

For the exercise sessions on 21 May 2026.

Exercise S12.1 – Integrality and Matching

We have already seen Frobenius' Theorem: *Let $k \geq 1$. Every k -regular bipartite graph contains a perfect matching.* The proof we have seen uses Hall's theorem. In this exercise, we want to find another proof using our knowledge about flows.

Let $G = (A \dot{\cup} B, E)$ be a k -regular graph for $k \geq 1$.

- (a) Describe how to model bipartite matching on G as a flow problem in a network $N = (V, A, c, s, t)$.
- (b) Construct explicitly a (not necessarily integer!) flow f on N with $\text{val}(f) = n$. Conclude that $\text{maxflow}(N) = n$.
Hint: you don't have to construct an integral flow. Make sure to define the flow value on all edges of your network N !
- (c) Using the result from (b), show that there exists an integer valued flow f' with $\text{val}(f') = n$. Using f' , prove that G contains a perfect matching M .