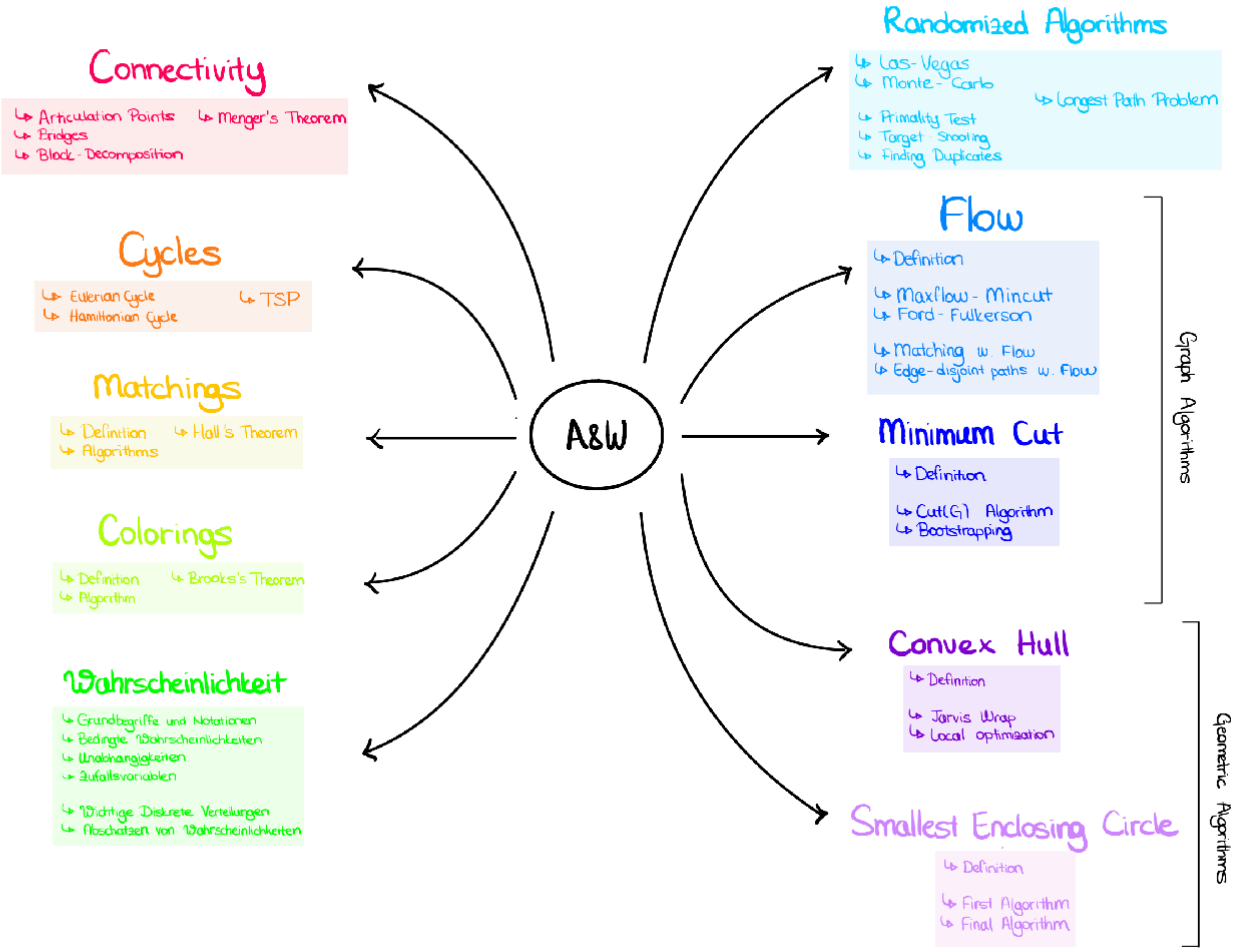
A&W

Extra Session

Flow + Colorful Paths

Nil Ozer

A&W Overview



A&W
Connectivity
Articulation Points
Bridges
Block-Decomposition
Menger's Theorem
Cycles
Eulerian Cycle
Hamiltonian Cycle
TSP
Matchings
Definition
Algorithms
Hall's Theorem
Colorings
Definition
Algorithm
Brooks's Theorem
Wahrscheinlichkeit
Grundbegriffe und Notationen
Bedingte Wahrscheinlichkeiten
Unabhangigkeiten
Zufallsvariablen
Wichtige Diskrete Verteilungen
Abschatzen von Wahrscheinlichkeiten
Randomized Algorithms
Las-Vegas
Monte-Carlo
Primality Test
Target-Shooting
Finding Duplicates
Longest Path Problem
Flow
Definition
Maxflow-Mincut
Ford-Fulkerson
Matching w. Flow
Edge-disjoint paths w. Flow
Minimum Cut
Definition
Cut(G) Algorithm
Bootstrapping
Graph Algorithms
Convex Hull
Definition
Jarvis Wrap
Local optimization
Smallest Enclosing Circle
Definition
First Algorithm
Final Algorithm
Geometric Algorithms

Last Weeks ...

- 15.05 extra session: Longest Path (Colorful Paths)
- 21.05 session : Minimum Cut, Smallest Enclosing Cycle
- **26.05 END OF SEMESTER, WHAT NOW? (18:00 - 21:00)**
- 28.05 session : Convex Hull
- Last extra session : Coding
    - worst case zoom

End of semester... What now?

Plan your summer Lernphase with the help of TAs from sem. 2

A&W	**Nil Özer**
A&W	**Victoria Miebach**
PProg	**Jennis Bešić**
DDCA	**Nicole Stadler**
Analysis I	**Mateo Zuluaga**

Sign up now!

CAB G 61 | 18:15-20:00

Followed by an Apéro

CSNOW

S11: Flows

Let's take a break

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CSNOW

Randomized Algorithms

Colorful Paths

Helper

Mathematical Tools and Notations

$[n] := \{1,2,\ldots,n\}$

$[n]^k :=$ the set of sequences over $[n]$ of length k $\quad |[n]^k| = n^k$

$\binom{[n]}{k} :=$ the set of k-element subsets of $[n]$ $\quad \left|\binom{[n]}{k}\right| = \binom{n}{k}$.

The k nodes on a path of length $k-1$ can be colored using $[k]$ in exactly k^k ways

$k!$ of these colorings use each color exactly once

Helper

Mathematical Tools and Notations

Handshaking lemma : For all graphs , it holds that $\sum_{v \in V} \deg(v) = 2|E|$.

If you repeat an experiment with success probability p until success, then the expected number of trials is $\frac{1}{p}$ ($Geo(p)$)

Helper

Mathematical Tools and Notations

For $c, n \in \mathbb{R}^+$, it holds that $c^{\log n} = n^{\log c}$

$$2^{\log n} = n^{\log 2} = n \text{ and } 2^{\mathcal{O}(\log n)} = n^{\mathcal{O}(1)} \text{ is always polynomial in } n$$

For $n \in \mathbb{N}_0$, it holds that $\sum_{i=0}^{n} \binom{n}{i} = 2^n$ (binomial theorem)

For $n \in \mathbb{N}_0$, it holds that $\frac{n!}{n^n} \geq e^{-n}$ (power series expansion of the exponential function)

Long-Path

Problem Description

given : A graph G and a number $B \in \mathbb{N}_0$

to find : is there a path of length B in G

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NP-Complete

Detour !

Colorful Paths

Problem Description

given : A graph $G = (V, E)$

A coloring of its vertices with k colors $\gamma : V \rightarrow [k]$

to find : Is there a colorful path of length $k - 1$ in a randomly colored graph ?

colorful :

A path is colorful if all of the vertices in the path have a different color

Colorful Paths

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A path is colorful if all of the vertices in the path have a different color

$P_i(v) := \{S \subseteq [k], |S| = i + 1 \ \mid$ There exists a colorful path of length i ending in v with colors $S\}$

$\exists$ colorful path of length $k - 1 \iff \bigcup_{v \in V} P_{k-1}(v) \neq \emptyset$

Colorful Paths

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- $P_1(v) = \{\{\gamma(x), \gamma(v)\} \mid x \in N(v)$ und $\gamma(x) \neq \gamma(v)\}$

Colorful Paths

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- $P_i(v) = \bigcup_{x \in N(v)} \{R \cup \{\gamma(v)\} \mid R \in P_{i-1}(x) \text{ und } \gamma(v) \notin R\}$

Colorful Paths

Algorithm

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Algorithm 1: COLORFUL(G, i) — *G a γ-colored graph*

forall $v \in V$ **do**
  $P_i(v) \leftarrow \emptyset$;
  forall $x \in N(v)$ **do**
    forall $R \in P_{i-1}(x)$ *such that* $\gamma(v) \notin R$ **do**
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

Algorithm 2: RAINBOW(G, γ) — *G a graph, γ a k-coloring*

forall $v \in V$ **do**
  $P_0(v) \leftarrow \{\{\gamma(v)\}\}$;
for $i = 1$ **to** $k-1$ **do**
  COLORFUL(G, i);
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Colorful Paths

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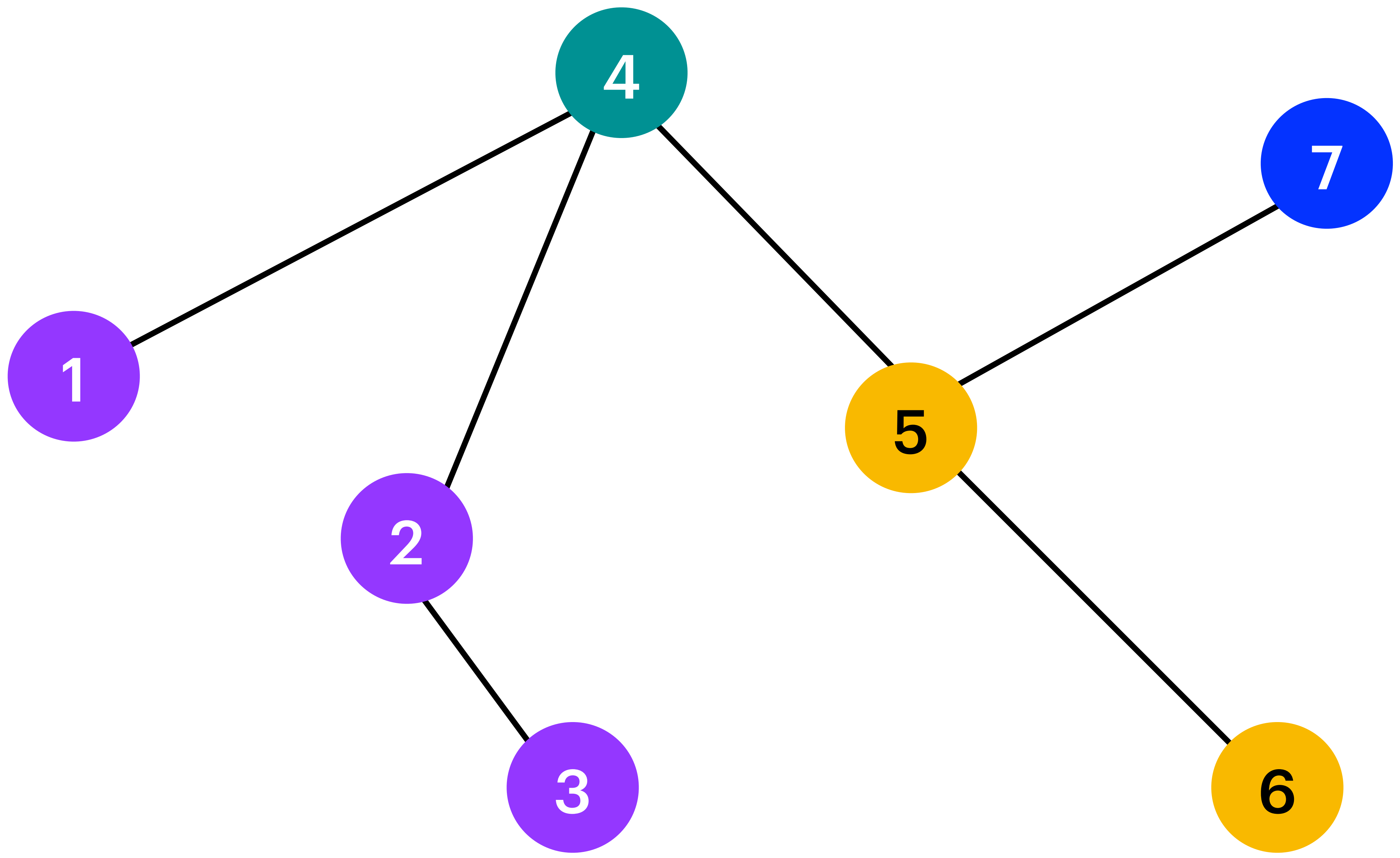
P_0	
P_0 (1)	
P_0 (2)	
P_0 (3)	
P_0 (4)	
P_0 (5)	
P_0 (6)	
P_0 (7)	



color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

γ 1 , 2 , 3 , 4

Colorful Paths
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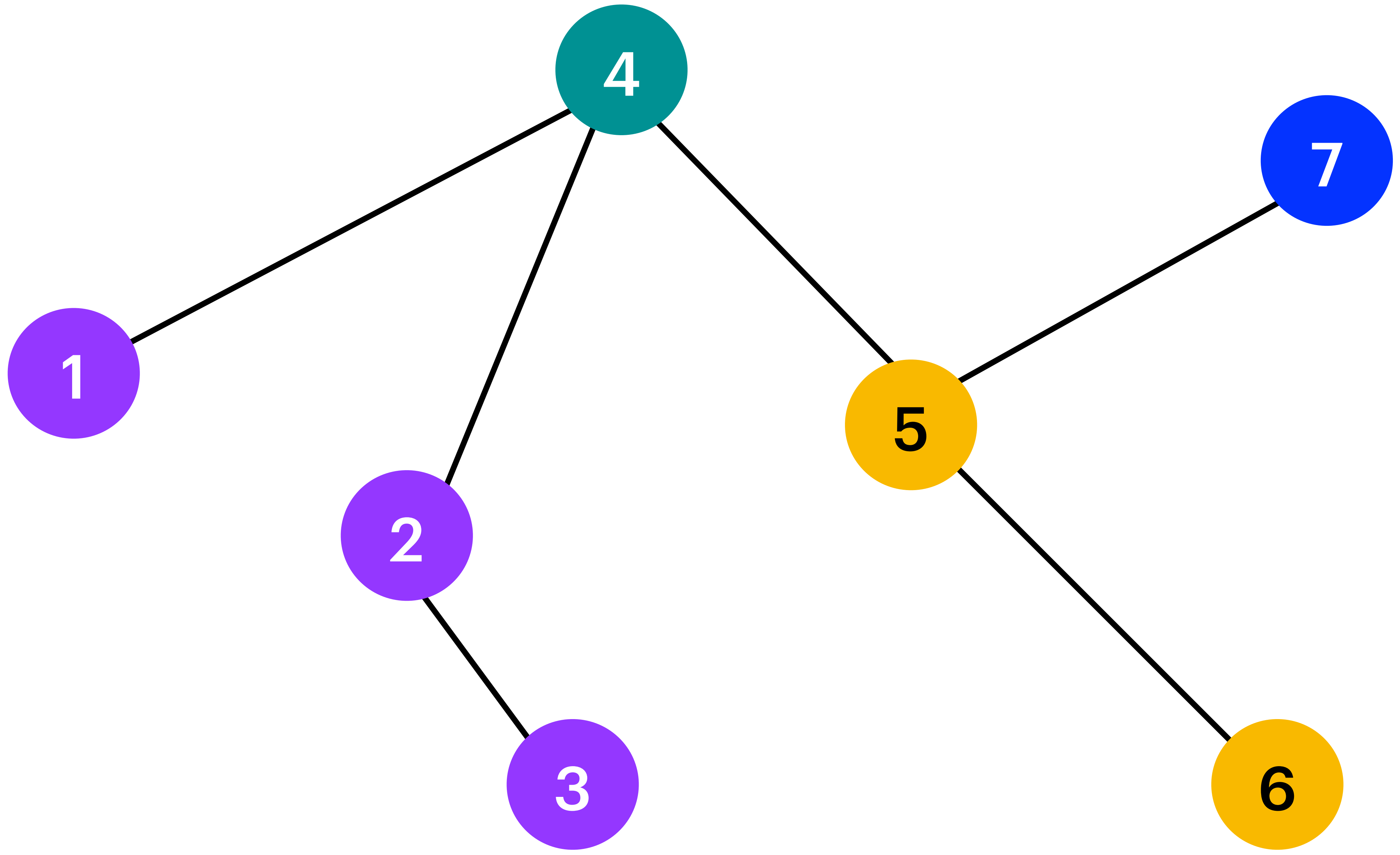
P0	
P0 (1)	{{1}}
P0 (2)	{{1}}
P0 (3)	{{1}}
P0 (4)	{{2}}
P0 (5)	{{3}
P0 (6)	{{3}}
P0 (7)	{{4}}



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$P_1(2)$	
$P_1(3)$	
$P_1(4)$	
$P_1(5)$	
$P_1(6)$	
$P_1(7)$	

P_0	
$P_0(1)$	{{1}}
$P_0(2)$	{{1}}
$P_0(3)$	{{1}}
$P_0(4)$	{{2}}
$P_0(5)$	{{3}
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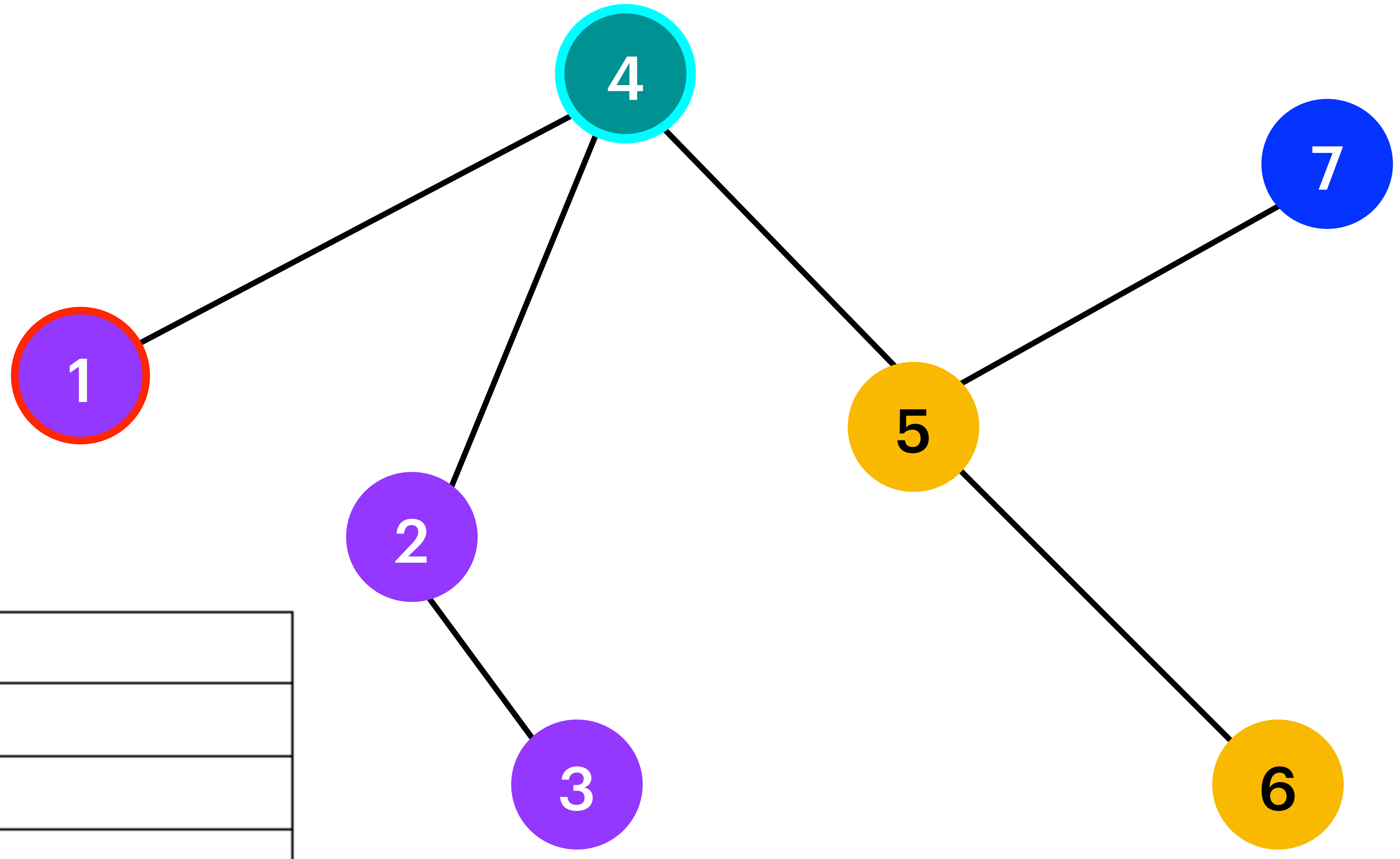

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$P_1(1)$	{{1 , 2}}
$P_1(2)$	
$P_1(3)$	
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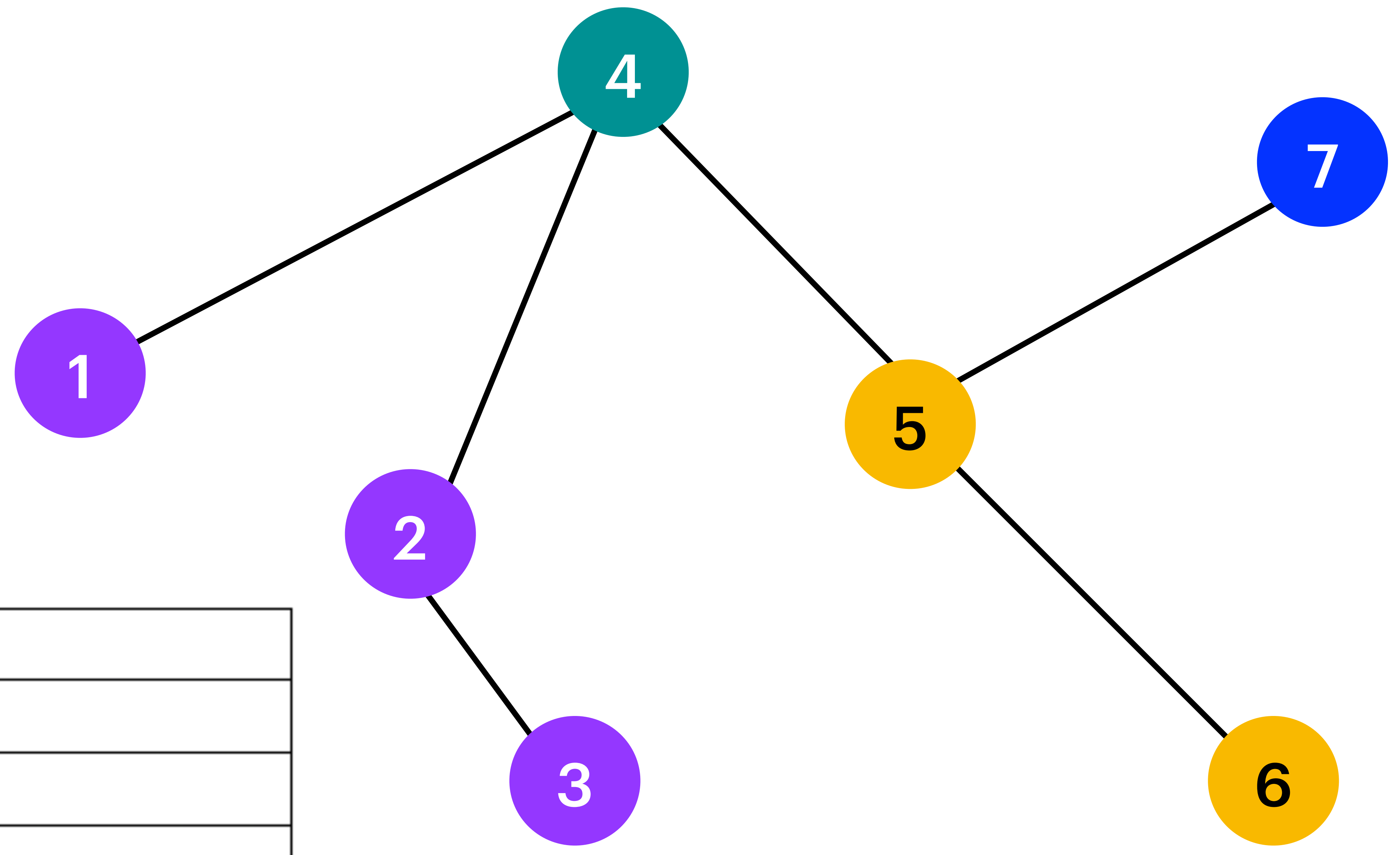

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$P_0(1)$	{{1}}
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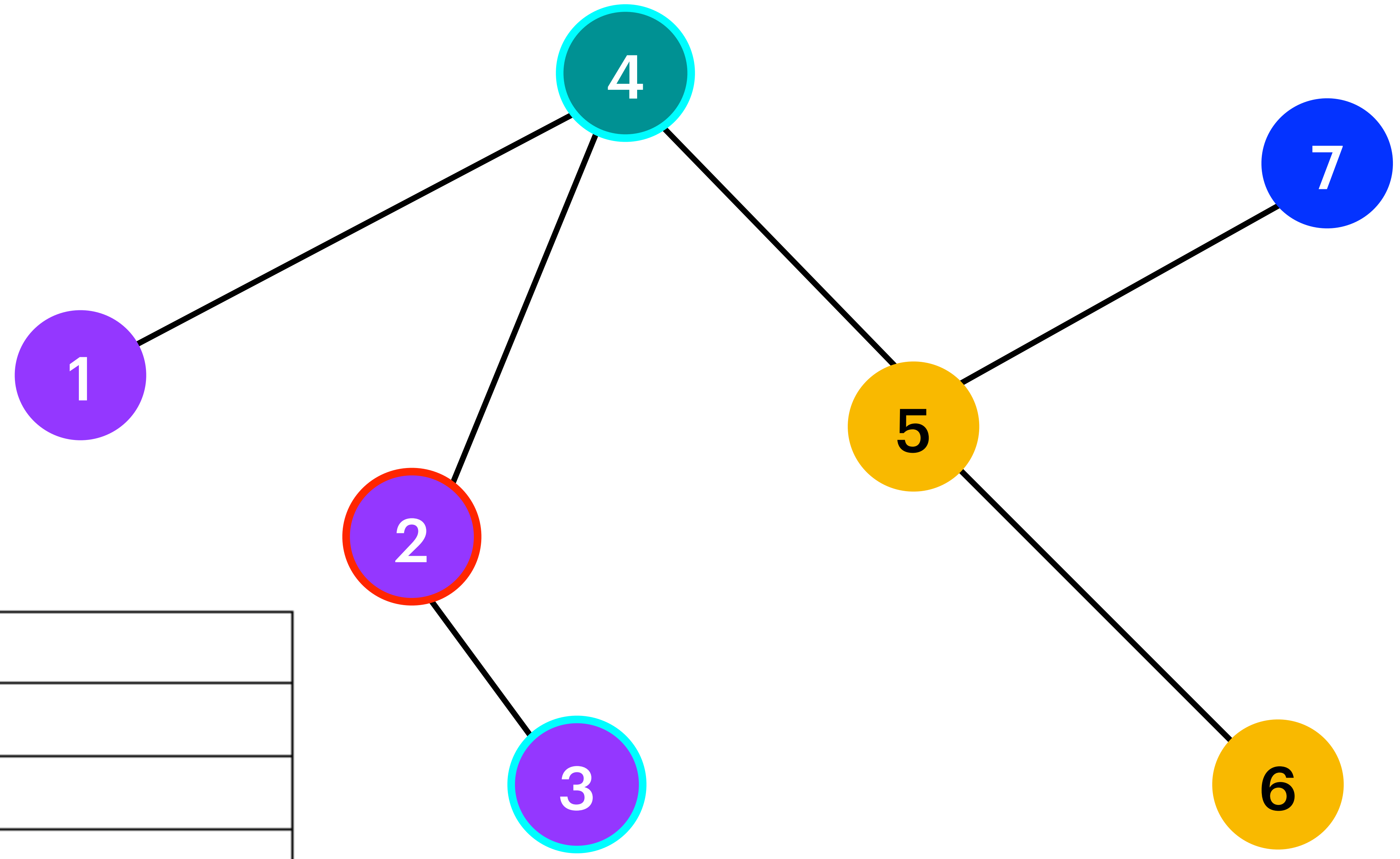

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$P_1(2)$	{{1 , 2}}
$P_1(3)$	
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P_0	
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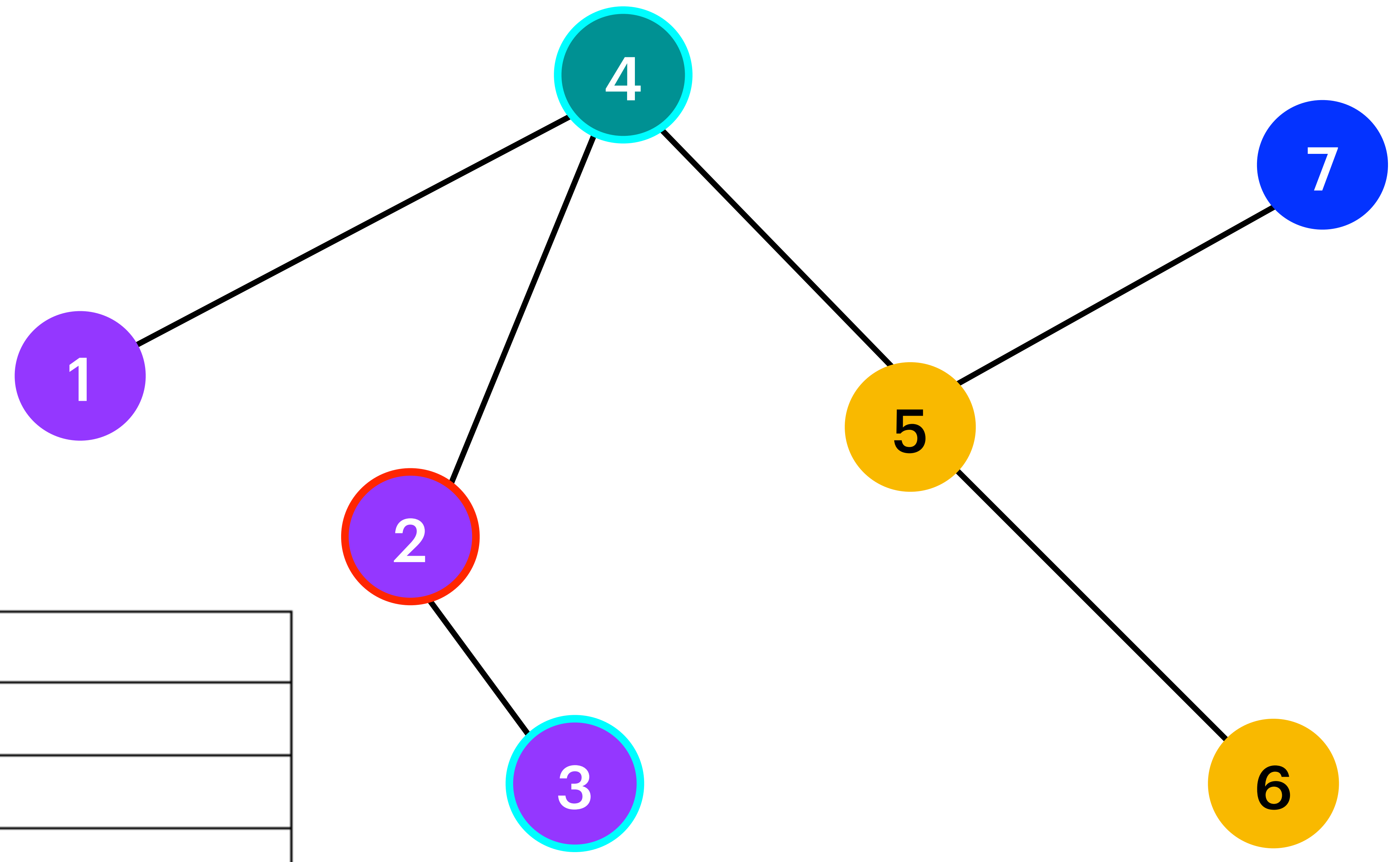

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P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	$\emptyset$
$P_1(4)$	
$P_1(5)$	
$P_1(6)$	
$P_1(7)$	

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$P_0(1)$	{{1}}
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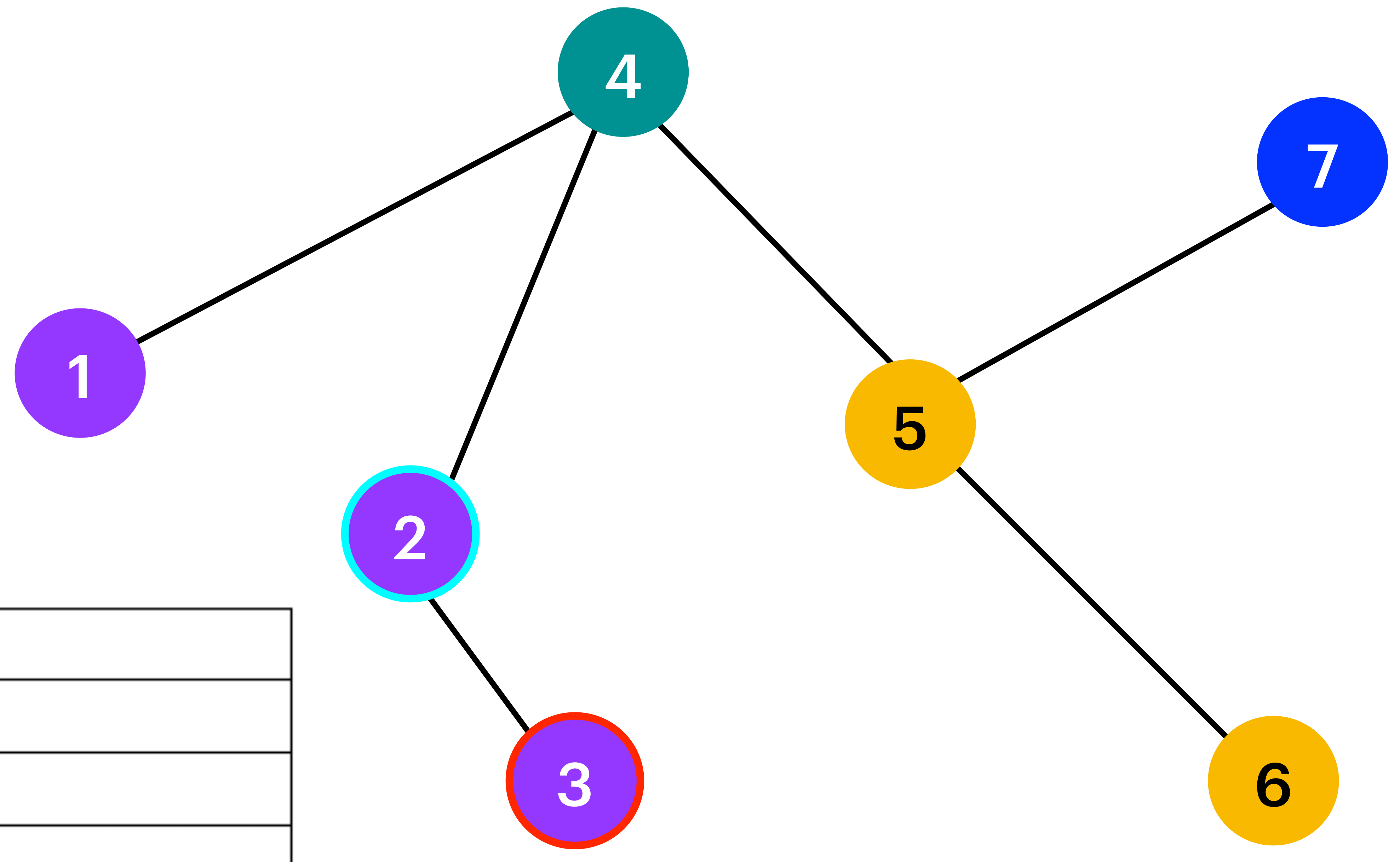
$P_0(3)$	{{1}}
$P_0(4)$	{{2}}
$P_0(5)$	{{3}
$P_0(6)$	{{3}}
$P_0(7)$	{{4}}



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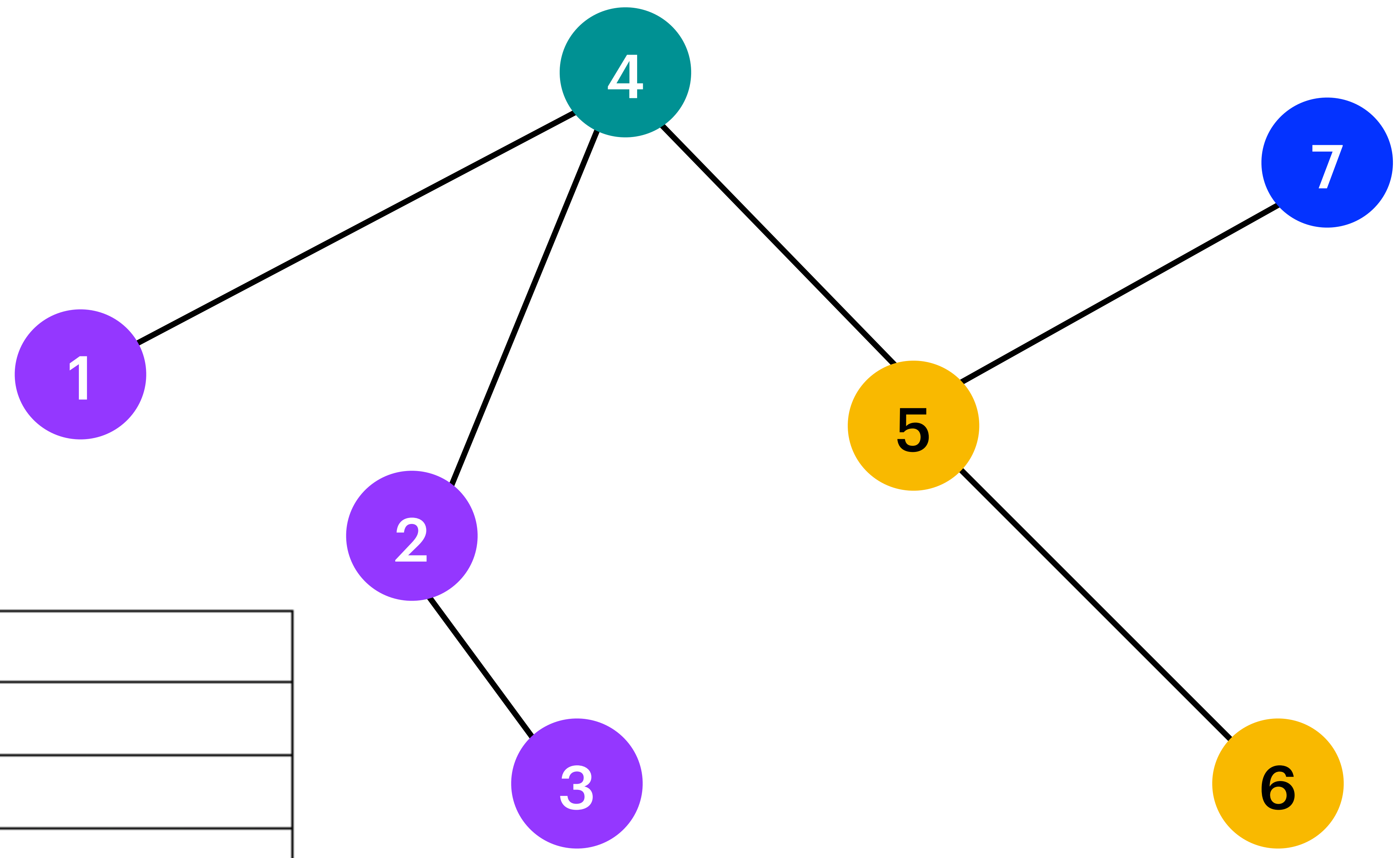
P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	
$P_1(6)$	
$P_1(7)$	

P_0	
$P_0(1)$	{{1}}
$P_0(2)$	{{1}}
$P_0(3)$	{{1}}
$P_0(4)$	{{2}}
$P_0(5)$	{{3}
$P_0(6)$	{{3}}
$P_0(7)$	{{4}}



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Colorful Paths

Algorithm

Algorithm 2: $\textsc{Rainbow}(G, \gamma)$

forall $v \in V$ **do**
  $P_0(v) \leftarrow \{\{\gamma(v)\}\}$;
for $i = 1$ **to** $k - 1$ **do**
  $\textsc{Colorful}(G, i)$;
return $\bigcup_{v \in V} P_{k-1}(v) \neq \emptyset$;

Algorithm 1: $\textsc{Colorful}(G, i)$

forall $v \in V$ **do**
  $P_i(v) \leftarrow \emptyset$;
  forall $x \in N(v)$ **do**
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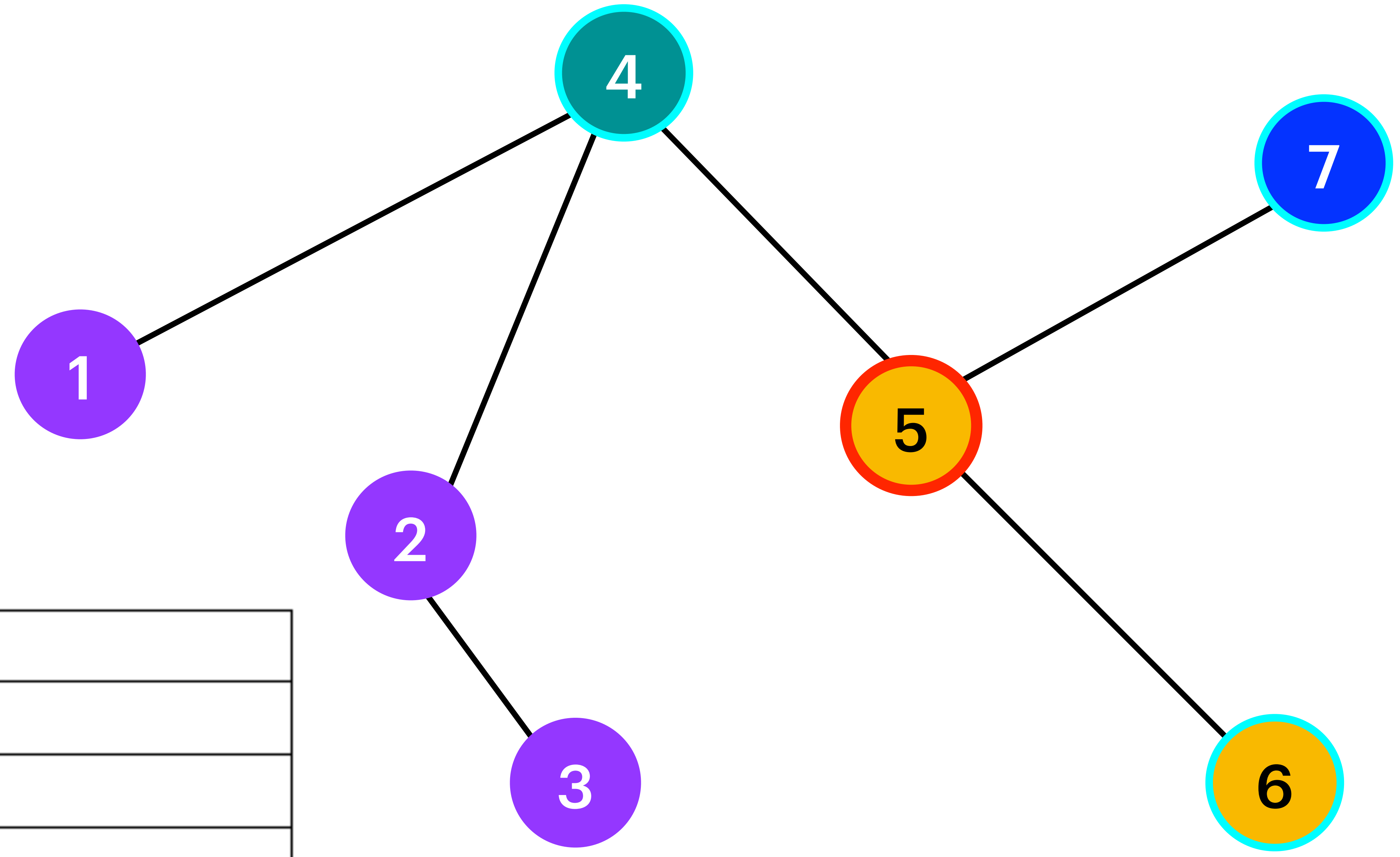
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	
$P_1(6)$	
$P_1(7)$	

P_0	
$P_0(1)$	{{1}}
$P_0(2)$	{{1}}
$P_0(3)$	{{1}}
$P_0(4)$	{{2}}
$P_0(5)$	{{3}
$P_0(6)$	{{3}}
$P_0(7)$	{{4}}



color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

γ 1 , 2 , 3 , 4

Colorful Paths

Algorithm

Algorithm 2: RAINBOW(G, γ)

forall $v \in V$ **do**
  $P_0(v) \leftarrow \{\{\gamma(v)\}\}$;
for $i = 1$ **to** $k - 1$ **do**
  COLORFUL(G, i);
return $\bigcup_{v \in V} P_{k-1}(v) \neq \emptyset$;

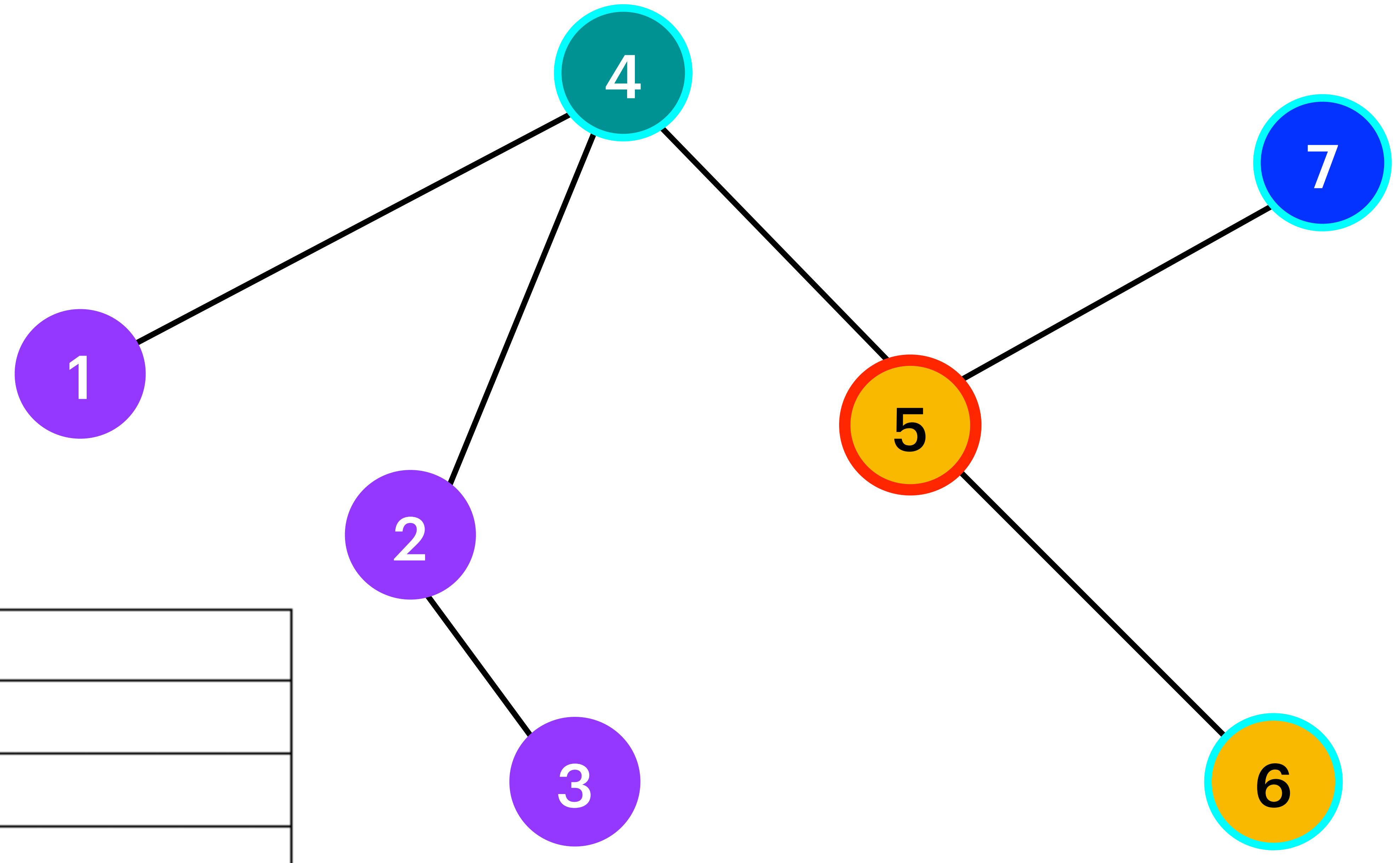
Algorithm 1: COLORFUL(G, i)

forall $v \in V$ **do**
  $P_i(v) \leftarrow \emptyset$;
  forall $x \in N(v)$ **do**
    forall $R \in P_{i-1}(x)$ *such that* $\gamma(v) \notin R$ **do**
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	$\varnothing$
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	
$P_1(7)$	

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

P_0	
$P_0(1)$	{{1}}
$P_0(2)$	{{1}}
$P_0(3)$	{{1}}
$P_0(4)$	{{2}}
$P_0(5)$	{{3}
$P_0(6)$	{{3}}
$P_0(7)$	{{4}}



γ 1 , 2 , 3 , 4

Colorful Paths

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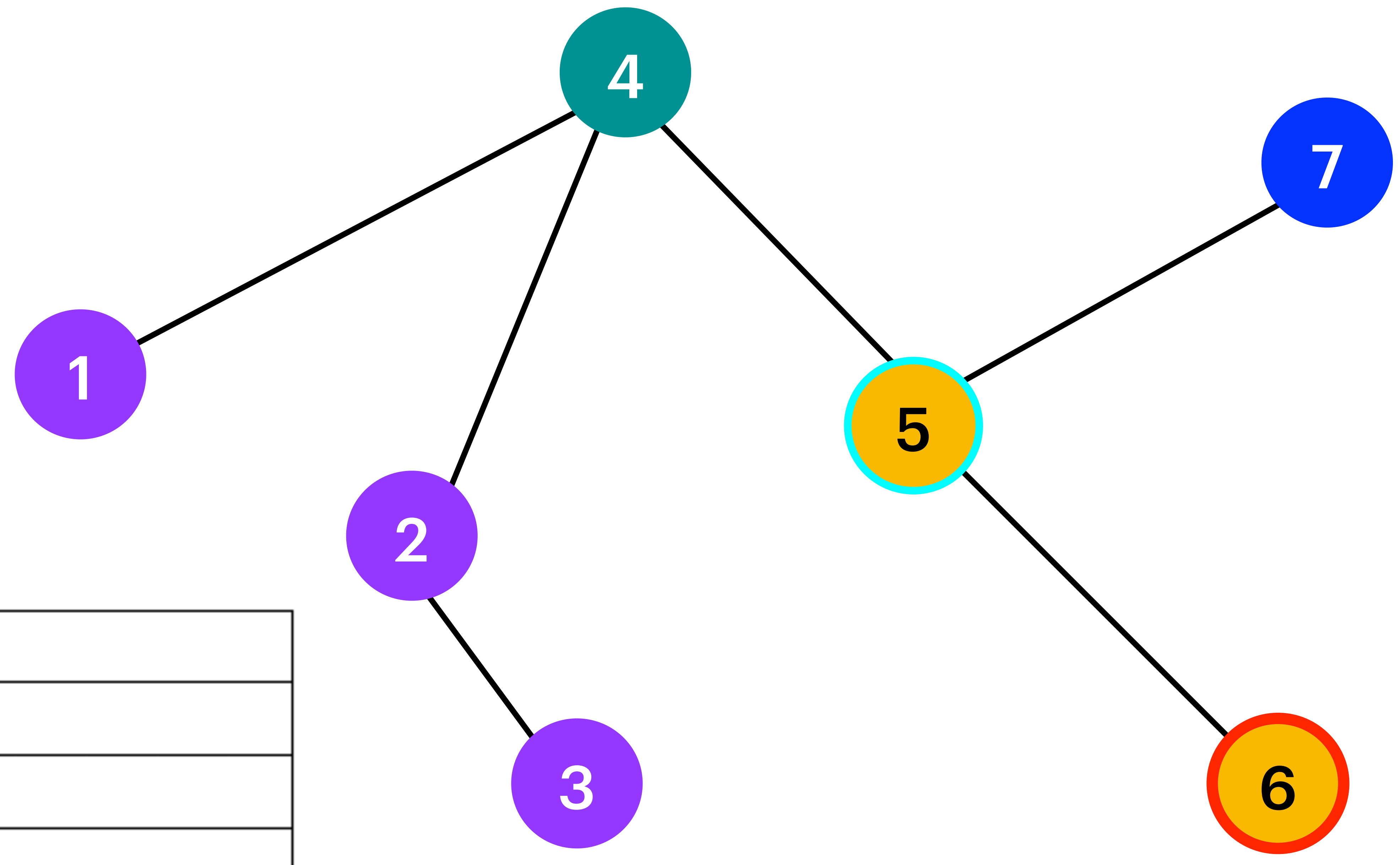
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	
$P_1(7)$	

P_0	
$P_0(1)$	{{1}}
$P_0(2)$	{{1}}
$P_0(3)$	{{1}}
$P_0(4)$	{{2}}
$P_0(5)$	{{3}
$P_0(6)$	{{3}}
$P_0(7)$	{{4}}



γ 1 , 2 , 3 , 4

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Colorful Paths

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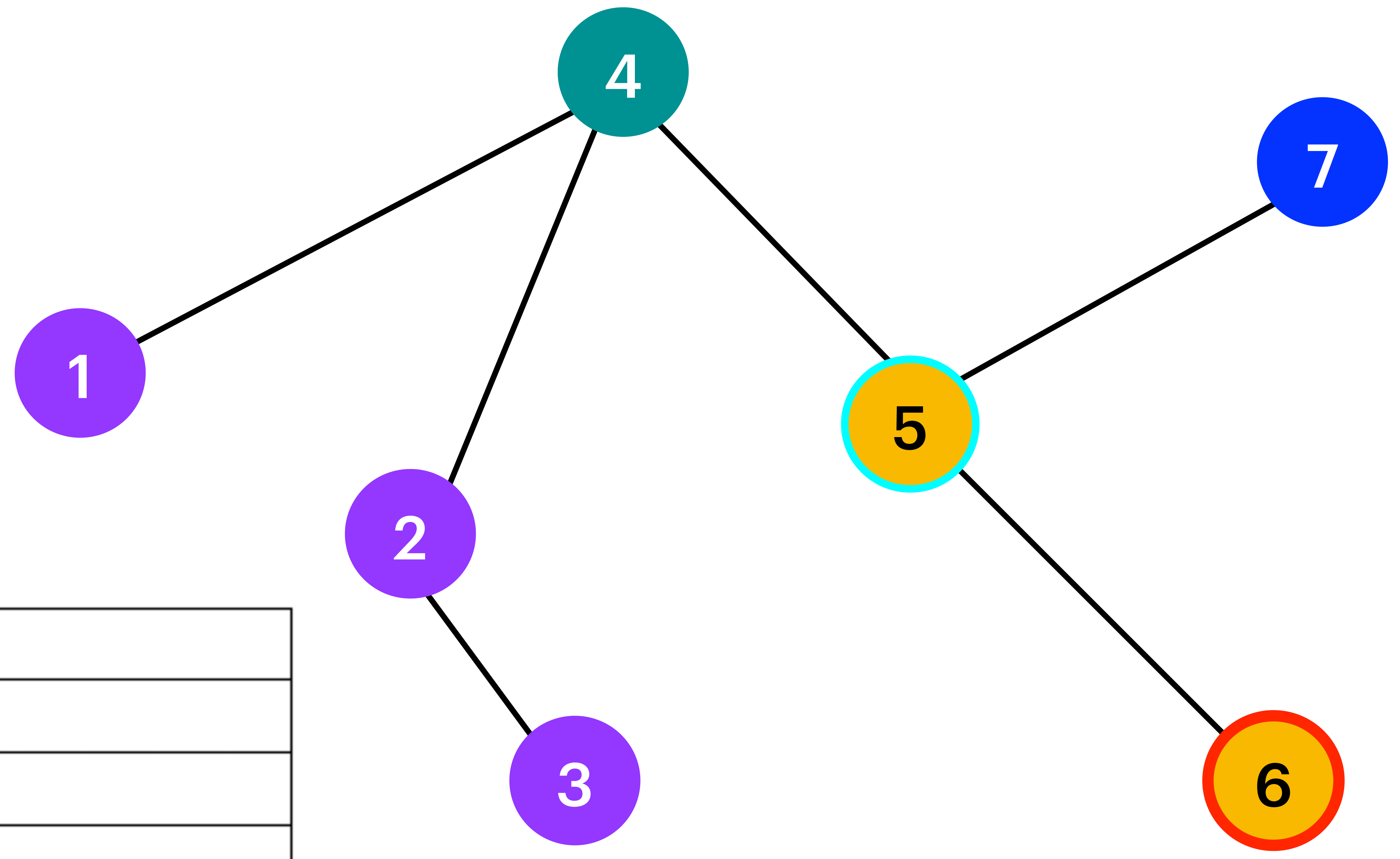
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	

P_0	
$P_0(1)$	{{1}}
$P_0(2)$	{{1}}
$P_0(3)$	{{1}}
$P_0(4)$	{{2}}
$P_0(5)$	{{3}
$P_0(6)$	{{3}}
$P_0(7)$	{{4}}



γ 1 , 2 , 3 , 4

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Colorful Paths

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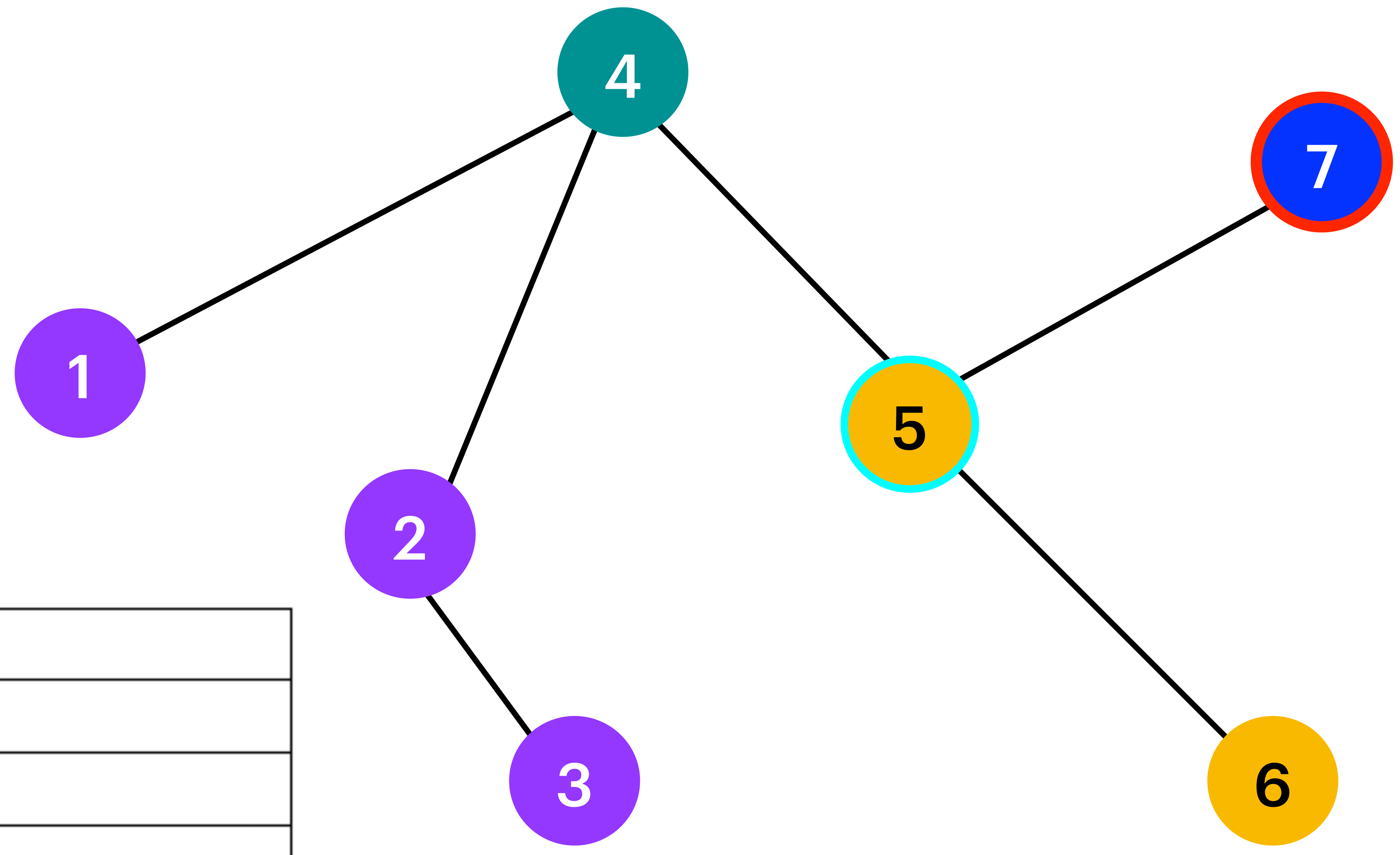
    forall $R \in P_{i-1}(x)$ *such that* $\gamma(v) \notin R$ **do**
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

P_0	
$P_0(1)$	{{1}}
$P_0(2)$	{{1}}
$P_0(3)$	{{1}}
$P_0(4)$	{{2}}
$P_0(5)$	{{3}
$P_0(6)$	{{3}}
$P_0(7)$	{{4}}



γ 1 , 2 , 3 , 4

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

Colorful Paths

Algorithm

Algorithm 2: RAINBOW(G, γ)

```
forall v ∈ V do
    P_0(v) ← {{γ(v)}};
for i = 1 to k − 1 do
    COLORFUL(G, i);
return ⋃_{v∈V} P_{k−1}(v) ≠ ∅;
```

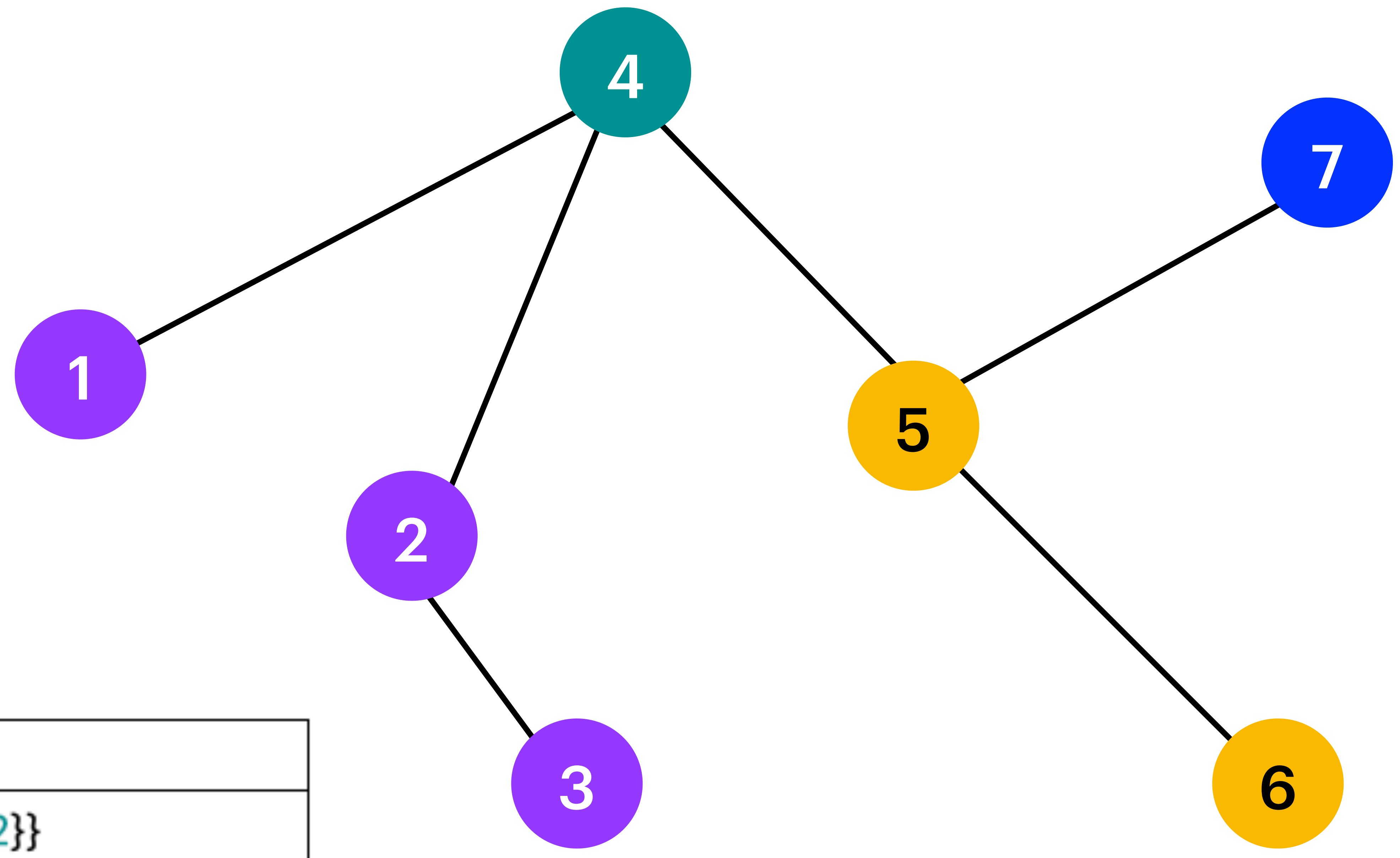
Algorithm 1: COLORFUL(G, i)

```
forall v ∈ V do
    P_i(v) ← ∅;
    forall x ∈ N(v) do
        forall R ∈ P_{i−1}(x) such that γ(v) ∉ R do
            P_i(v) ← P_i(v) ∪ {R ∪ {γ(v)}};
```

P_2	
$P_2(1)$	
$P_2(2)$	
$P_2(3)$	
$P_2(4)$	
$P_2(5)$	
$P_2(6)$	
$P_2(7)$	

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}



γ 1 , 2 , 3 , 4

Colorful Paths

Algorithm

Algorithm 2: RAINBOW(G, γ)

forall $v \in V$ **do**
  $P_0(v) \leftarrow \{\{\gamma(v)\}\}$;
for $i = 1$ **to** $k - 1$ **do**
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Algorithm 1: COLORFUL(G, i)

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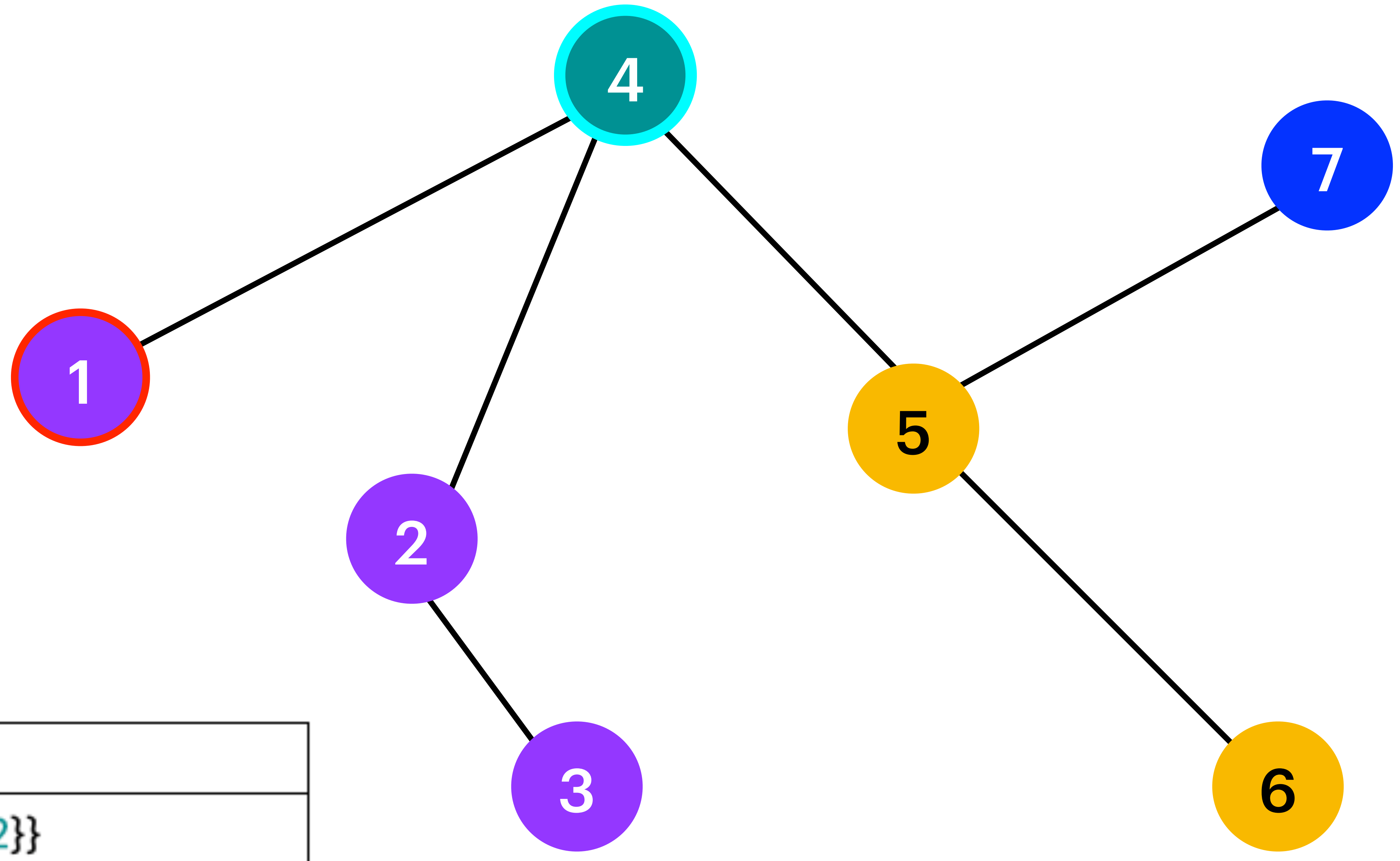
    forall $R \in P_{i-1}(x)$ *such that* $\gamma(v) \notin R$ **do**
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_2	
$P_2(1)$	
$P_2(2)$	
$P_2(3)$	
$P_2(4)$	
$P_2(5)$	
$P_2(6)$	
$P_2(7)$	

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S



P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

γ 1 , 2 , 3 , 4

Colorful Paths
Algorithm

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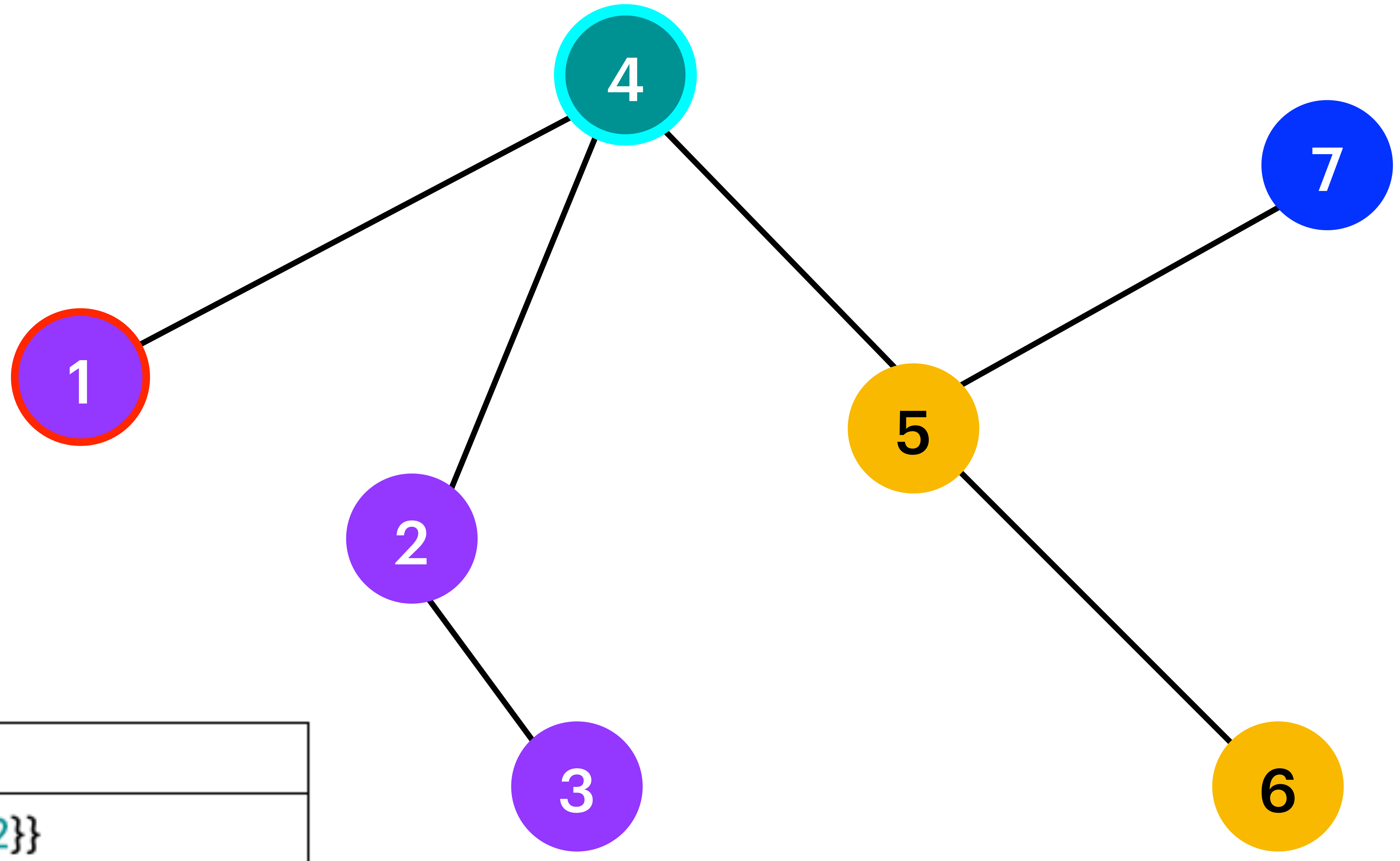

P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	
$P_2(3)$	
$P_2(4)$	
$P_2(5)$	
$P_2(6)$	
$P_2(7)$	

P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

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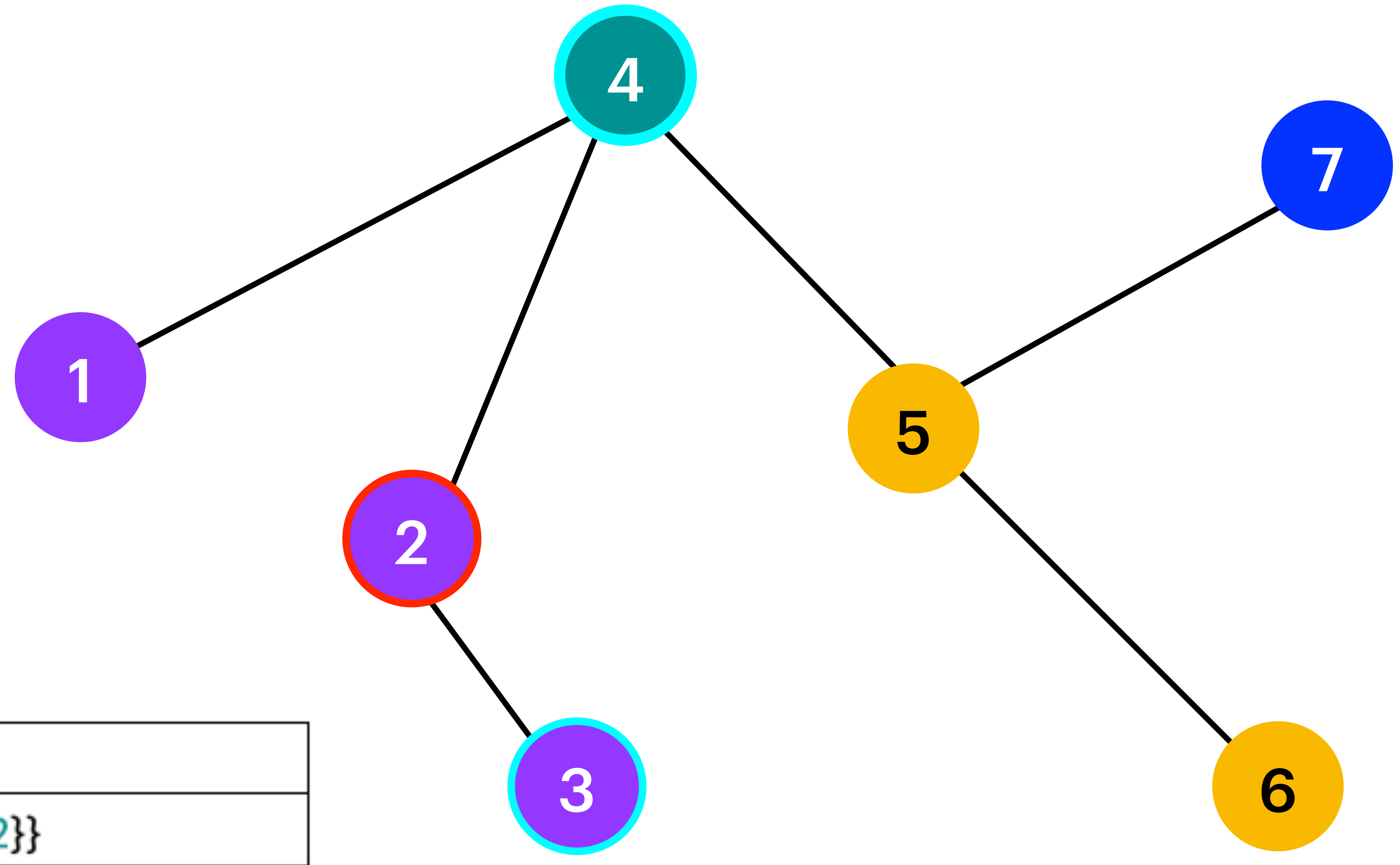

P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	
$P_2(3)$	
$P_2(4)$	
$P_2(5)$	
$P_2(6)$	
$P_2(7)$	

P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}

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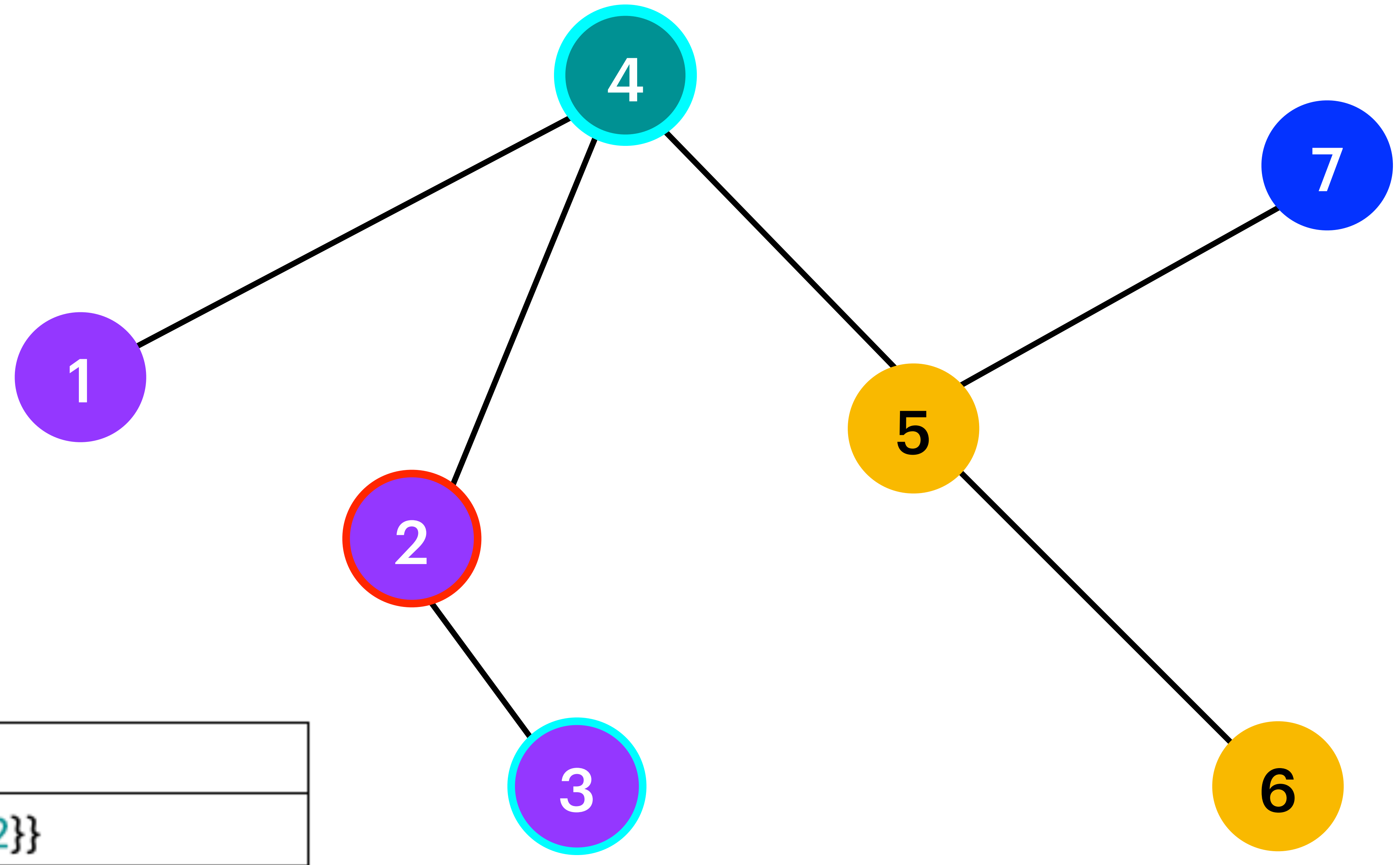
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	
$P_2(4)$	
$P_2(5)$	
$P_2(6)$	
$P_2(7)$	

P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}



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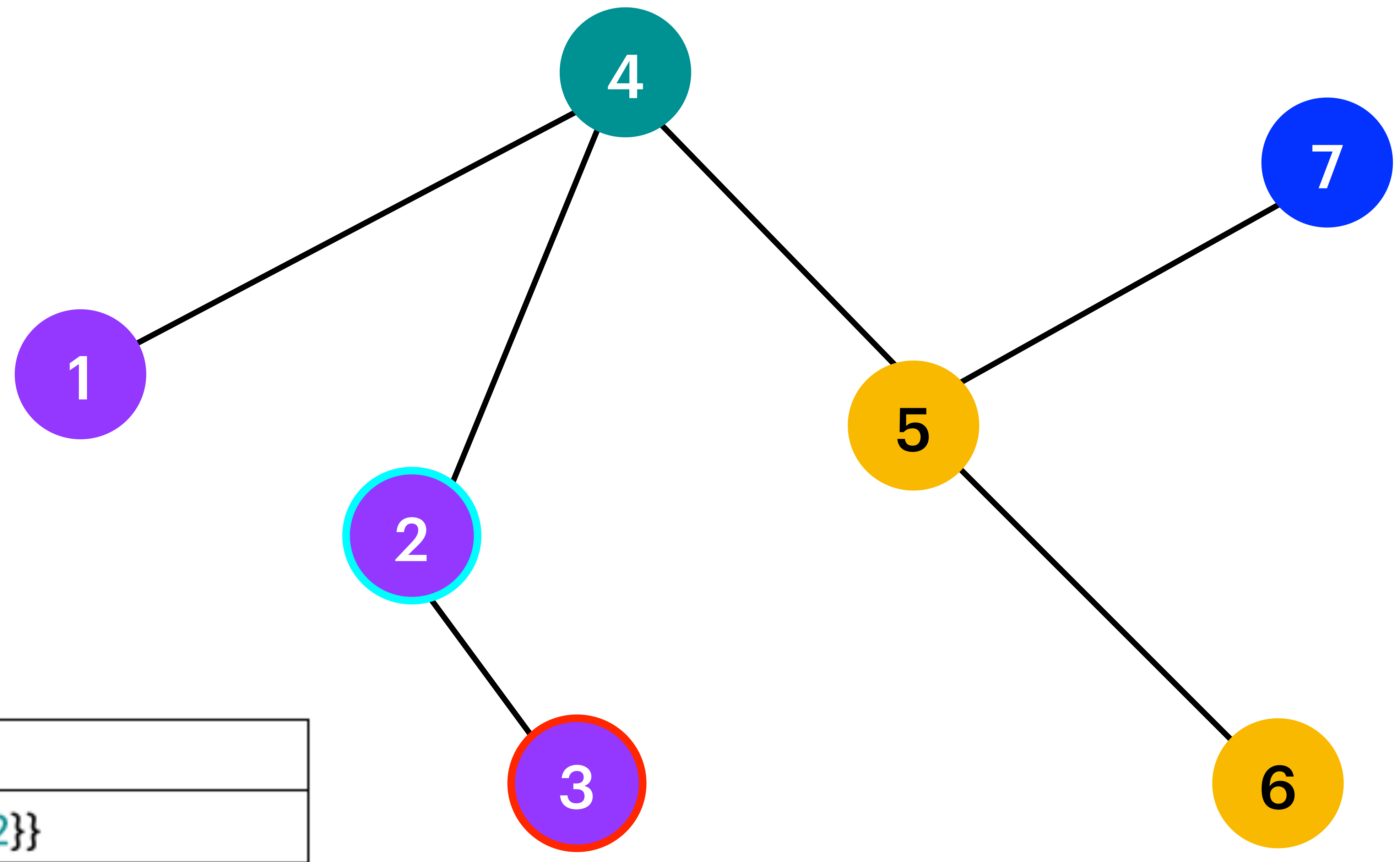
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	
$P_2(4)$	
$P_2(5)$	
$P_2(6)$	
$P_2(7)$	



P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

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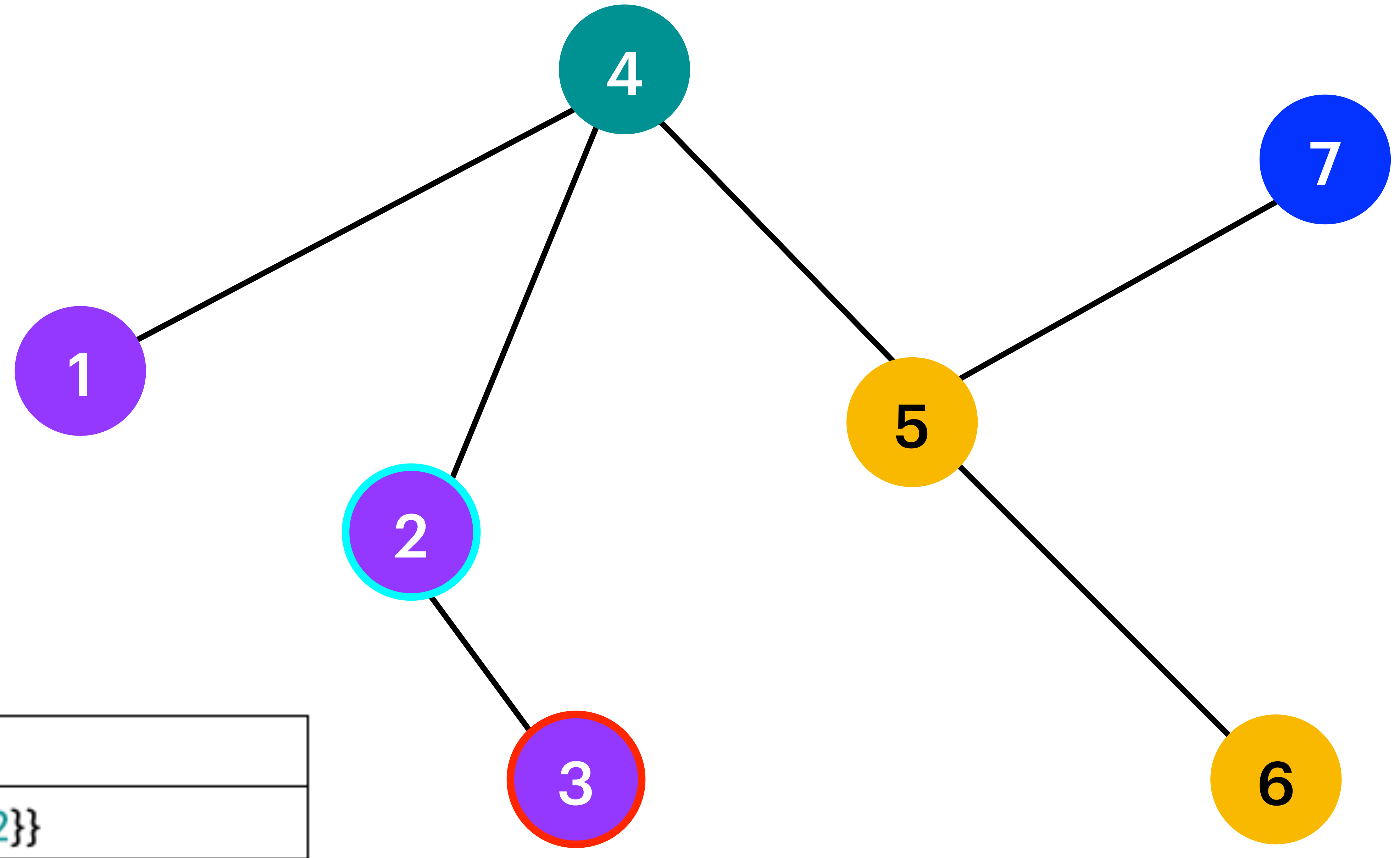
P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	
$P_2(5)$	
$P_2(6)$	
$P_2(7)$	



P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

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P_2	
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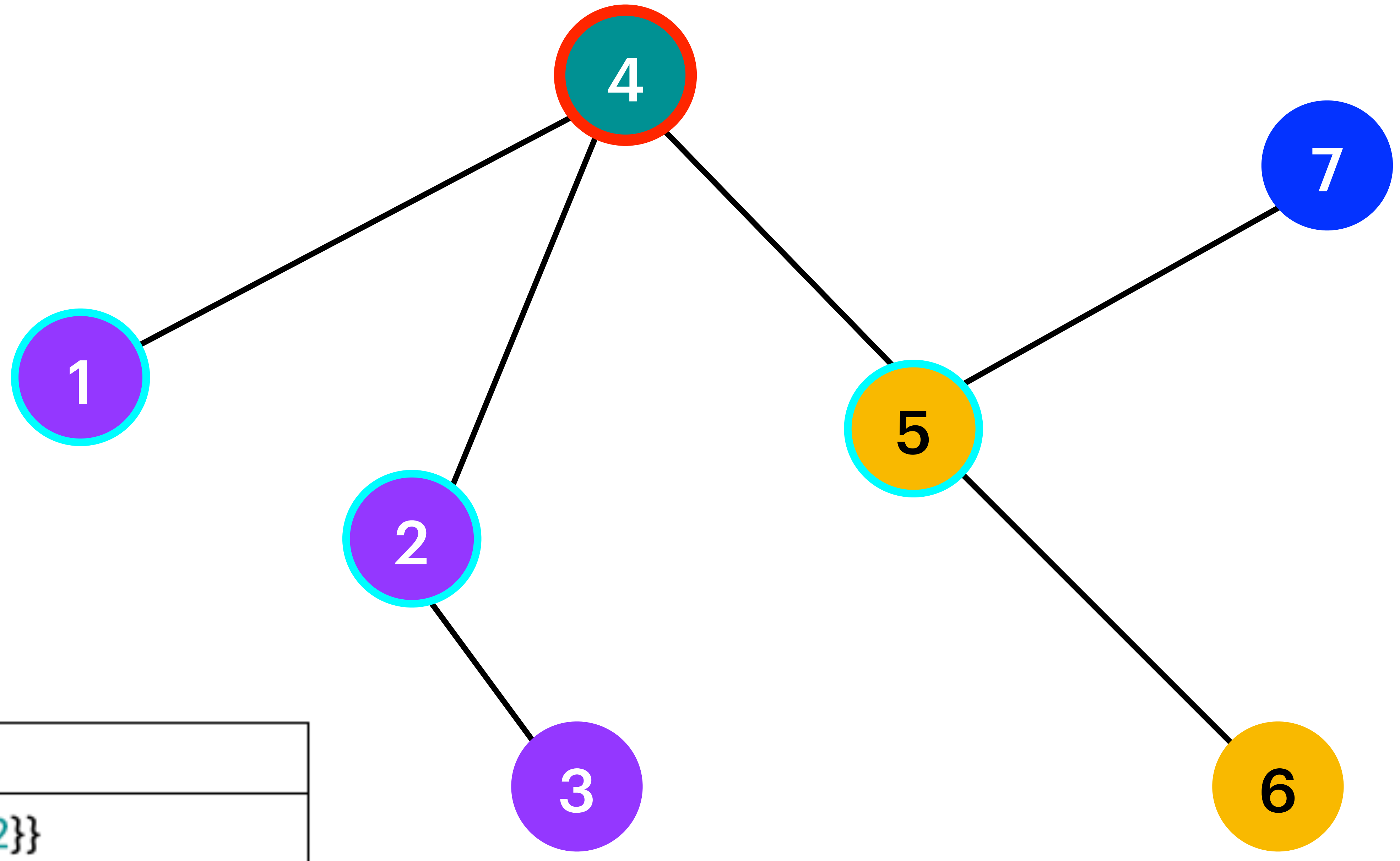
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	
$P_2(5)$	
$P_2(6)$	
$P_2(7)$	



P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
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$P_1(7)$	{{3 , 4}}}

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γ 1 , 2 , 3 , 4

Colorful Paths

Algorithm

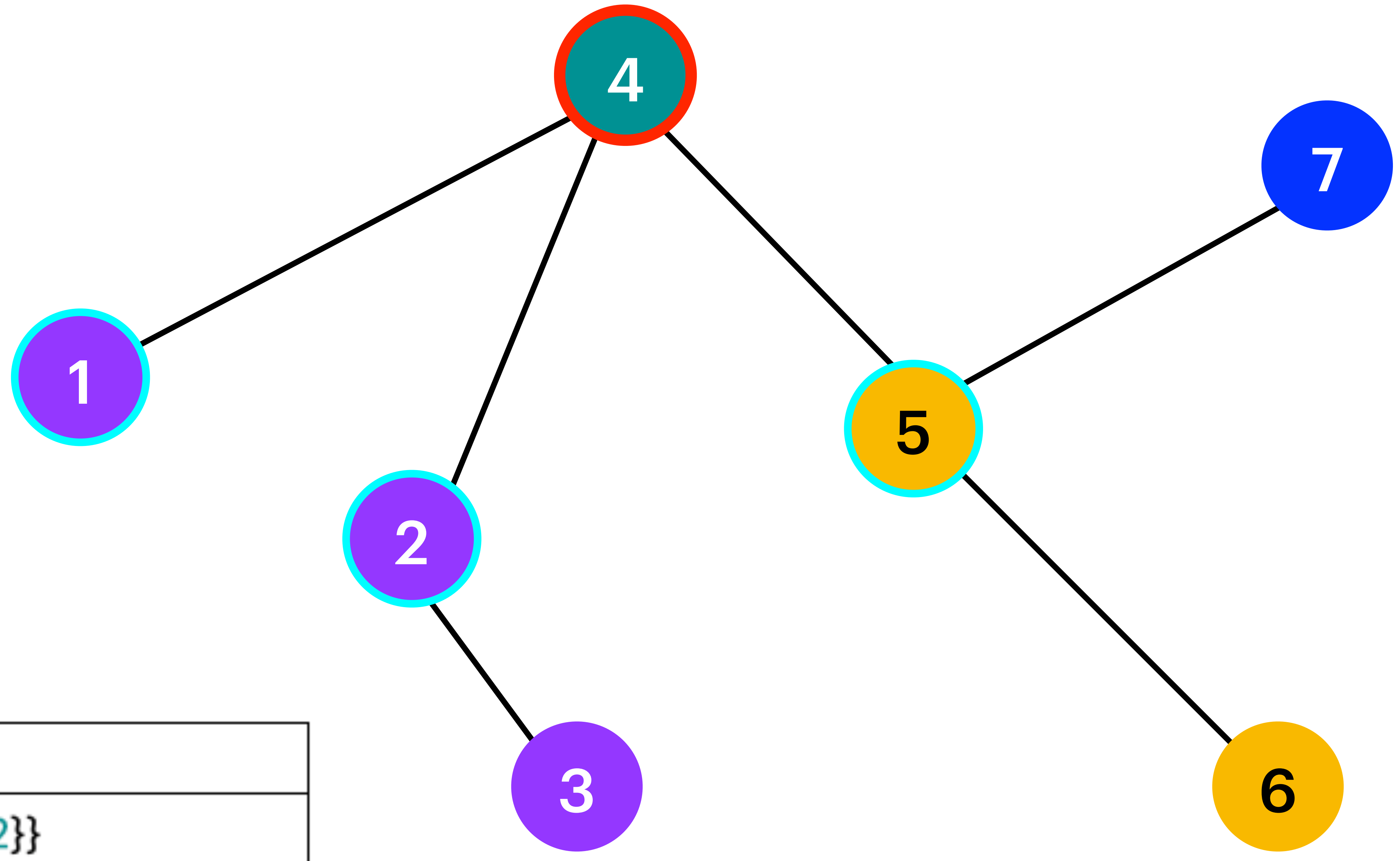
Algorithm 2: RAINBOW(G, γ)

```
forall v ∈ V do
    P_0(v) ← {{γ(v)}};
for i = 1 to k − 1 do
    COLORFUL(G, i);
return ⋃_{v∈V} P_{k−1}(v) ≠ ∅;
```

Algorithm 1: COLORFUL(G, i)

```
forall v ∈ V do
    P_i(v) ← ∅;
    forall x ∈ N(v) do
        forall R ∈ P_{i−1}(x) such that γ(v) ∉ R do
            P_i(v) ← P_i(v) ∪ {R ∪ {γ(v)}};
```

P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	
$P_2(6)$	
$P_2(7)$	



P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

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γ 1 , 2 , 3 , 4

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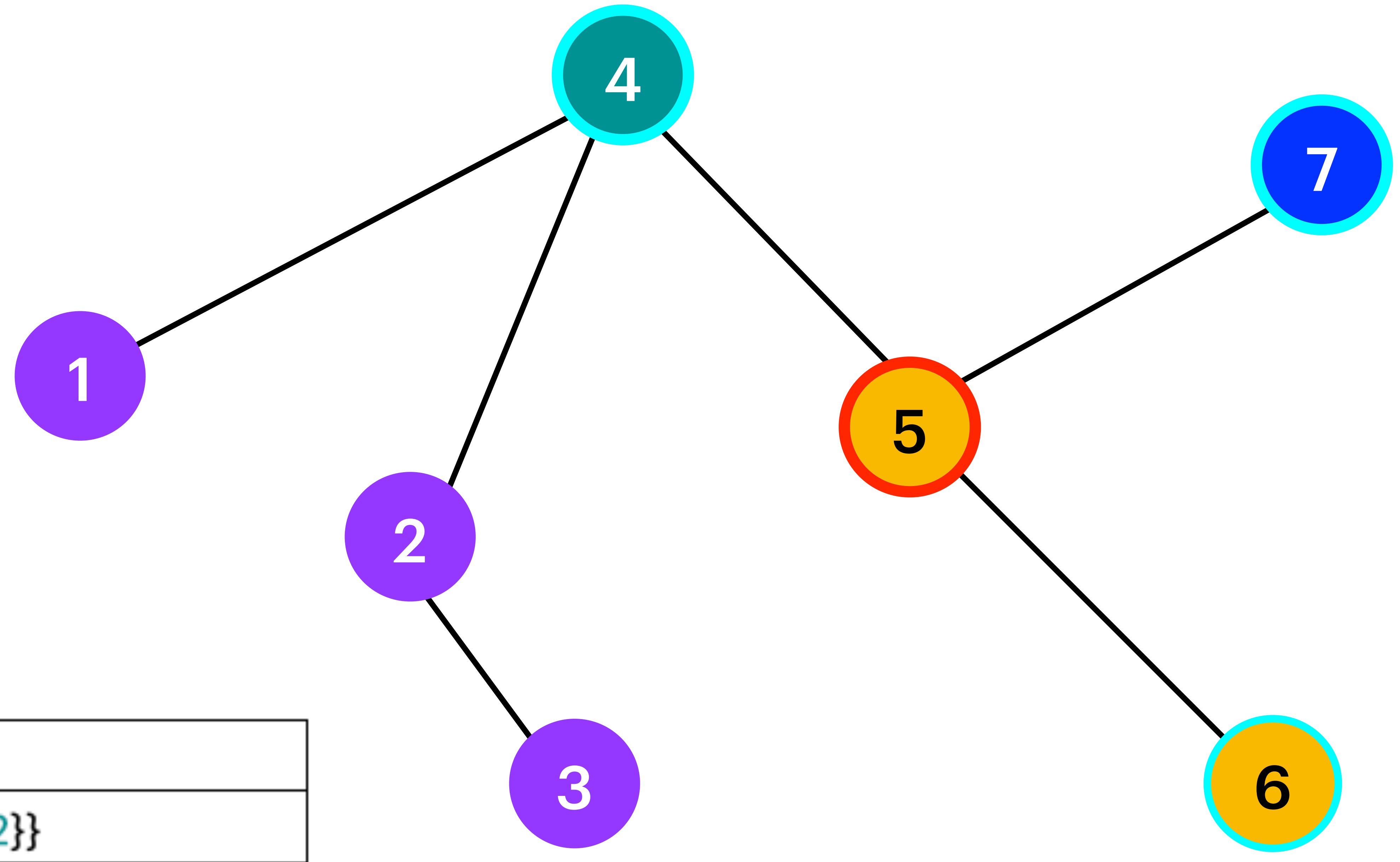
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	
$P_2(6)$	
$P_2(7)$	



P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

γ 1 , 2 , 3 , 4

Colorful Paths

Algorithm

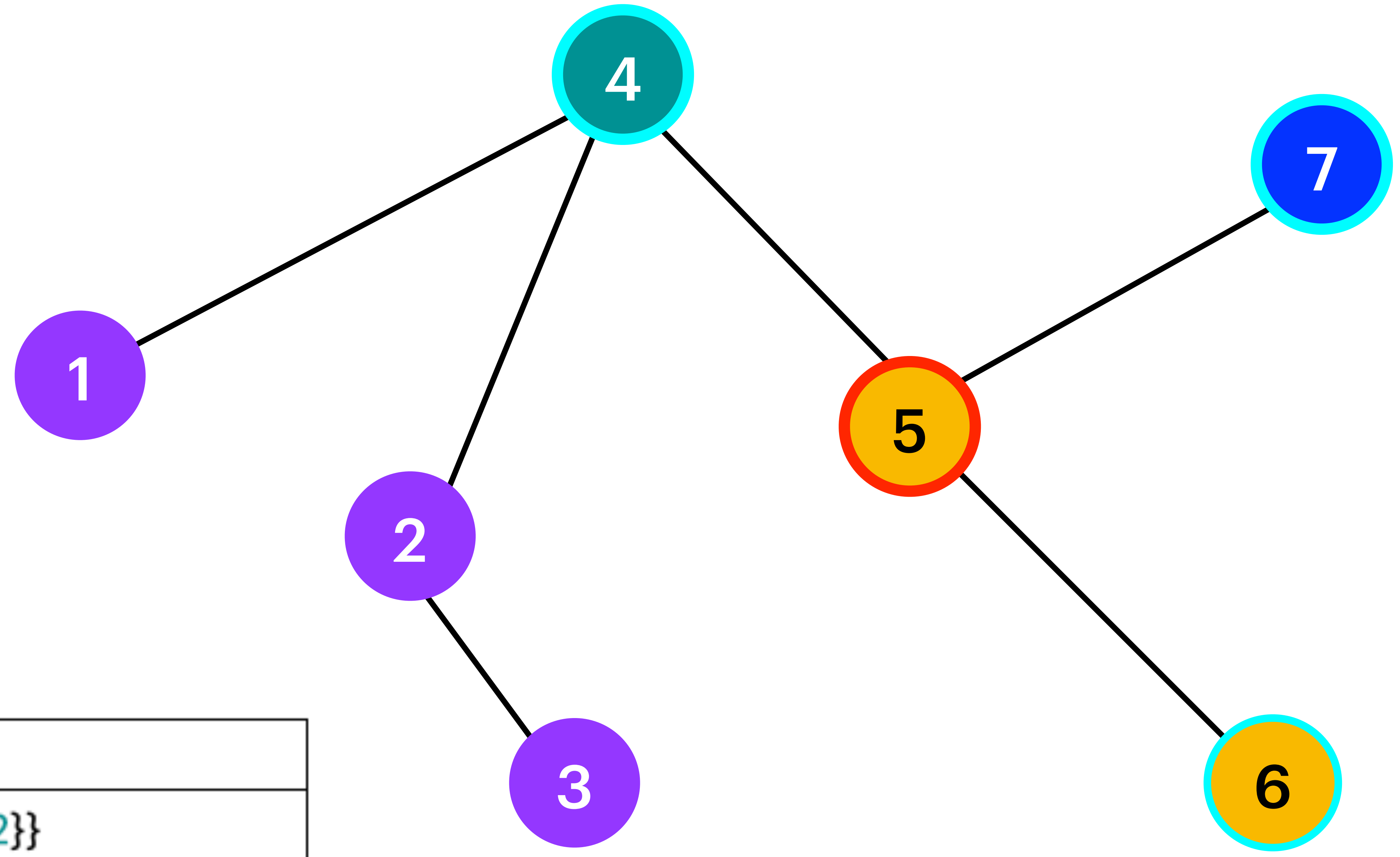
Algorithm 2: RAINBOW(G, γ)

forall $v \in V$ **do**
  $P_0(v) \leftarrow \{\{\gamma(v)\}\}$;
for $i = 1$ **to** $k - 1$ **do**
  COLORFUL(G, i);
return $\bigcup_{v \in V} P_{k-1}(v) \neq \emptyset$;

Algorithm 1: COLORFUL(G, i)

forall $v \in V$ **do**
  $P_i(v) \leftarrow \emptyset$;
  forall $x \in N(v)$ **do**
    forall $R \in P_{i-1}(x)$ *such that* $\gamma(v) \notin R$ **do**
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	{{1 , 2 , 3}}
$P_2(6)$	
$P_2(7)$	



P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

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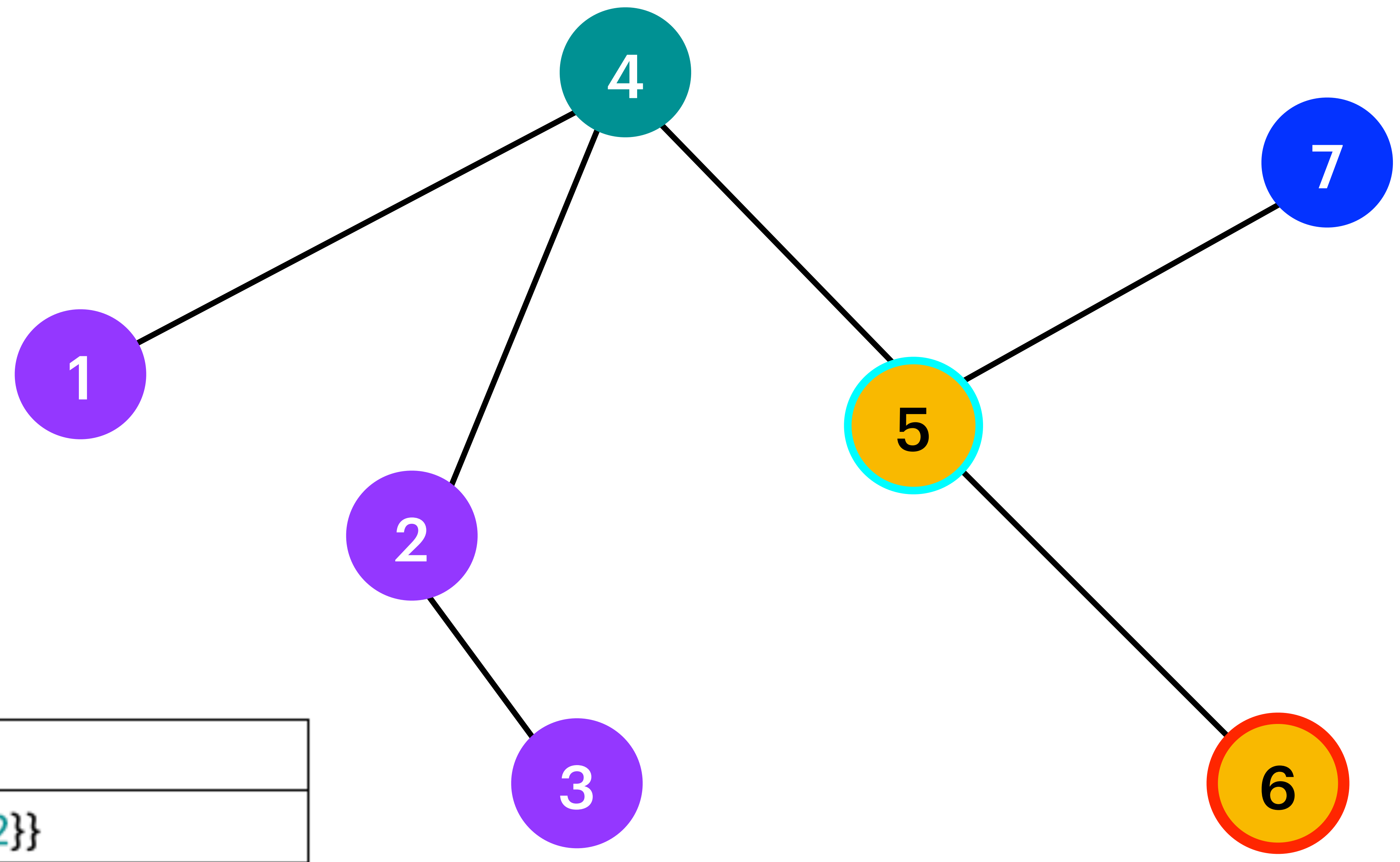
P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	{{1 , 2 , 3}}
$P_2(6)$	
$P_2(7)$	



P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

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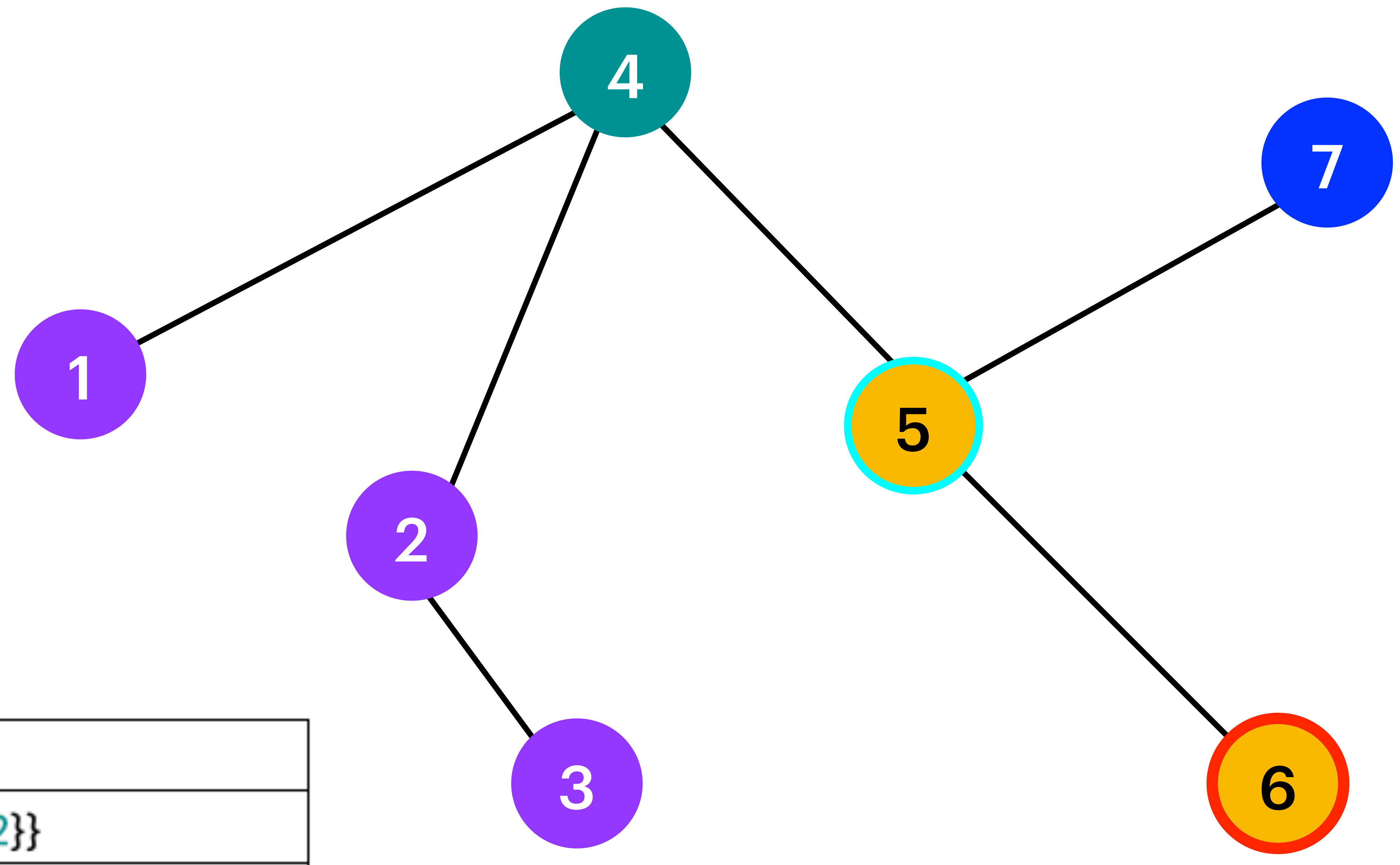
P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	{{1 , 2 , 3}}
$P_2(6)$	∅
$P_2(7)$	



P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

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Colorful Paths

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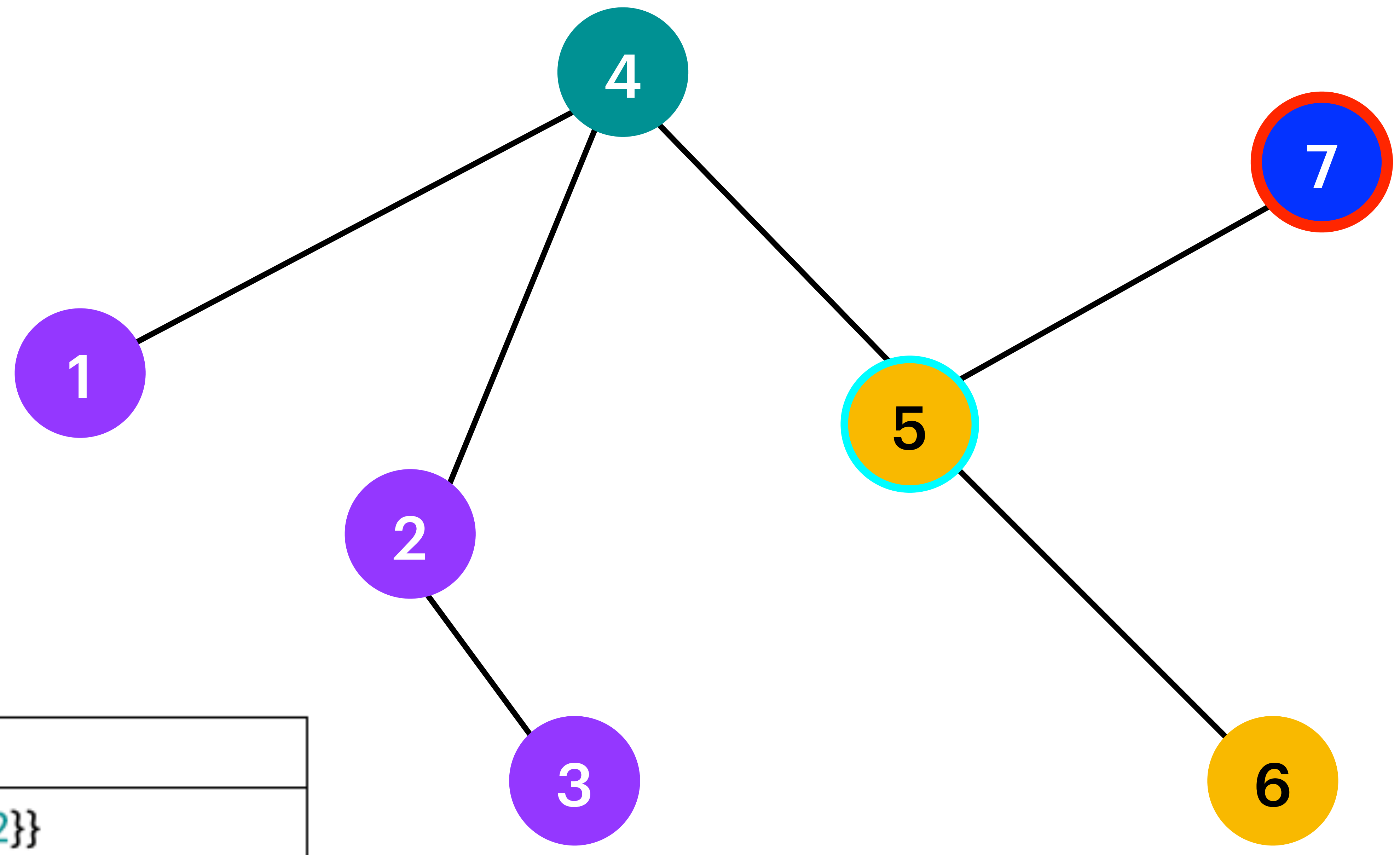
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	{{1 , 2 , 3}}
$P_2(6)$	∅
$P_2(7)$	{{2 , 3 , 4}}



P_1	
$P_1(1)$	{{1 , 2}}
$P_1(2)$	{{1 , 2}}
$P_1(3)$	∅
$P_1(4)$	{{1 , 2} , {2 , 3}}
$P_1(5)$	{{2 , 3} , {3 , 4}}
$P_1(6)$	∅
$P_1(7)$	{{3 , 4}}}

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

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Colorful Paths

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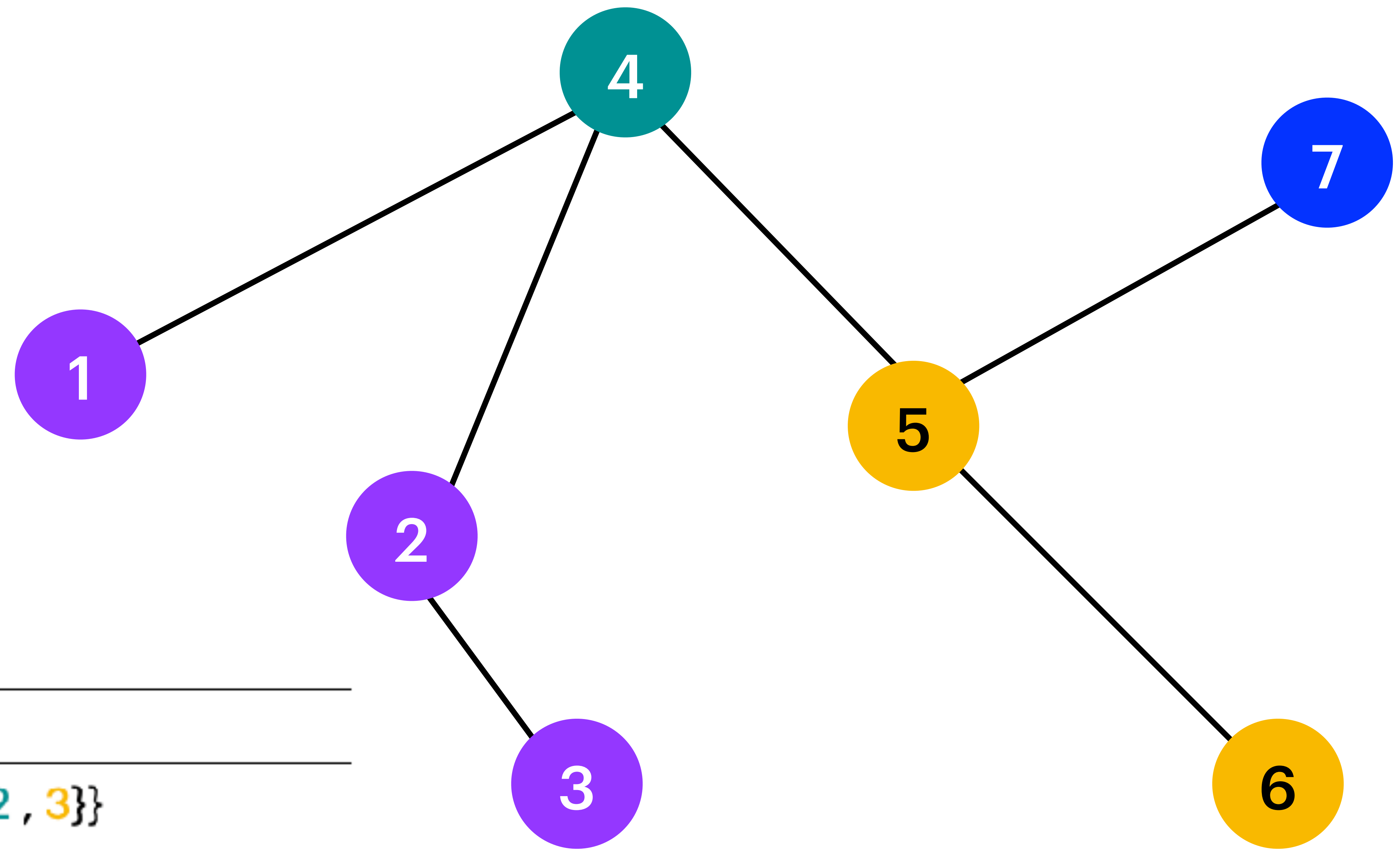
forall $v \in V$ **do**
  $P_i(v) \leftarrow \emptyset$;
  forall $x \in N(v)$ **do**
    forall $R \in P_{i-1}(x)$ *such that* $\gamma(v) \notin R$ **do**
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_3	
$P_3(1)$	
$P_3(2)$	
$P_3(3)$	
$P_3(4)$	
$P_3(5)$	
$P_3(6)$	
$P_3(7)$	



P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	{{1 , 2 , 3}}
$P_2(6)$	∅
$P_2(7)$	{{2 , 3 , 4}}

γ 1 , 2 , 3 , 4

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

Colorful Paths

Algorithm

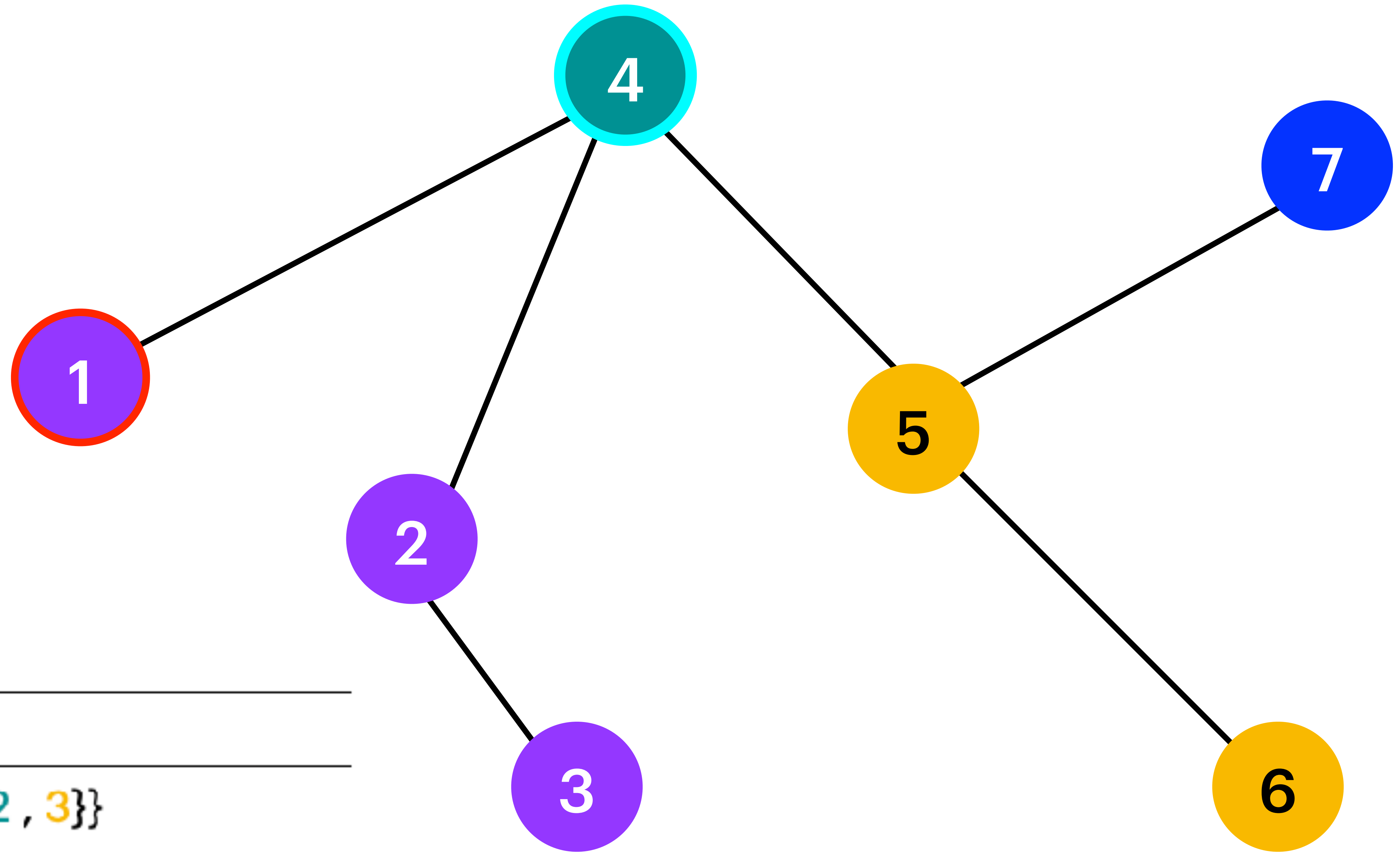
Algorithm 2: RAINBOW(G, γ)

```
forall v ∈ V do
    P_0(v) ← {{γ(v)}};
for i = 1 to k − 1 do
    COLORFUL(G, i);
return ⋃_{v∈V} P_{k−1}(v) ≠ ∅;
```

Algorithm 1: COLORFUL(G, i)

```
forall v ∈ V do
    P_i(v) ← ∅;
    forall x ∈ N(v) do
        forall R ∈ P_{i−1}(x) such that γ(v) ∉ R do
            P_i(v) ← P_i(v) ∪ {R ∪ {γ(v)}};
```

P_3	
$P_3(1)$	
$P_3(2)$	
$P_3(3)$	
$P_3(4)$	
$P_3(5)$	
$P_3(6)$	
$P_3(7)$	



P_2	
$P_2(1)$	{{1, 2, 3}}
$P_2(2)$	{{1, 2, 3}}
$P_2(3)$	∅
$P_2(4)$	{{2, 3, 4}}
$P_2(5)$	{{1, 2, 3}}
$P_2(6)$	∅
$P_2(7)$	{{2, 3, 4}}

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

γ 1, 2, 3, 4

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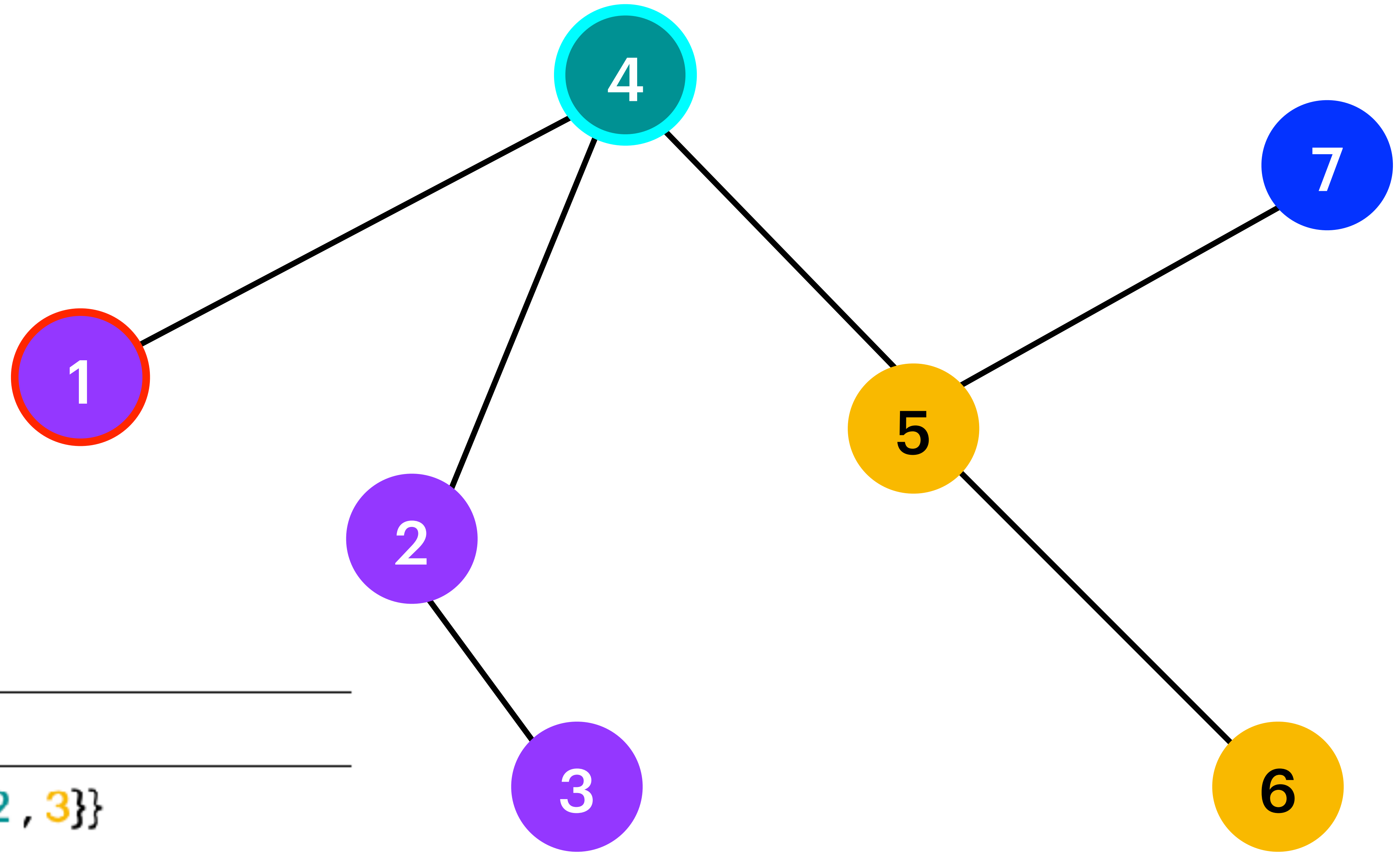
            $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_3	
$P_3(1)$	{{1 , 2 , 3 , 4}}
$P_3(2)$	
$P_3(3)$	
$P_3(4)$	
$P_3(5)$	
$P_3(6)$	
$P_3(7)$	



P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	{{1 , 2 , 3}}
$P_2(6)$	∅
$P_2(7)$	{{2 , 3 , 4}}

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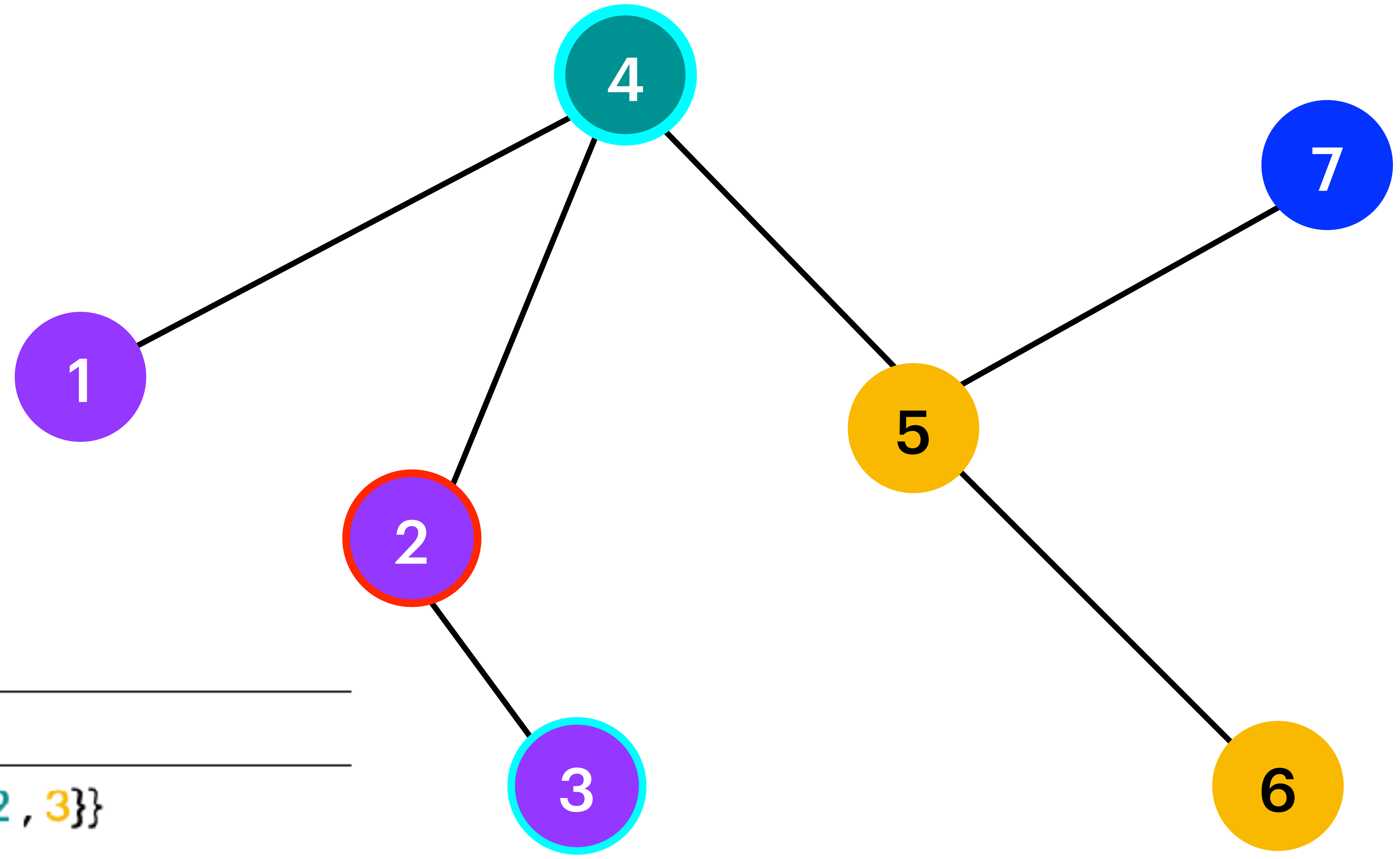
```

Algorithm 1: COLORFUL(G, i)

```
forall v ∈ V do
    P_i(v) ← ∅;
    forall x ∈ N(v) do
        forall R ∈ P_{i−1}(x) such that γ(v) ∉ R do
            P_i(v) ← P_i(v) ∪ {R ∪ {γ(v)}};
```

P_3	
$P_3(1)$	$\{\{1, 2, 3, 4\}\}$
$P_3(2)$	
$P_3(3)$	
$P_3(4)$	
$P_3(5)$	
$P_3(6)$	
$P_3(7)$	



P_2	
$P_2(1)$	$\{\{1, 2, 3\}\}$
$P_2(2)$	$\{\{1, 2, 3\}\}$
$P_2(3)$	$\emptyset$
$P_2(4)$	$\{\{2, 3, 4\}\}$
$P_2(5)$	$\{\{1, 2, 3\}\}$
$P_2(6)$	$\emptyset$
$P_2(7)$	$\{\{2, 3, 4\}\}$

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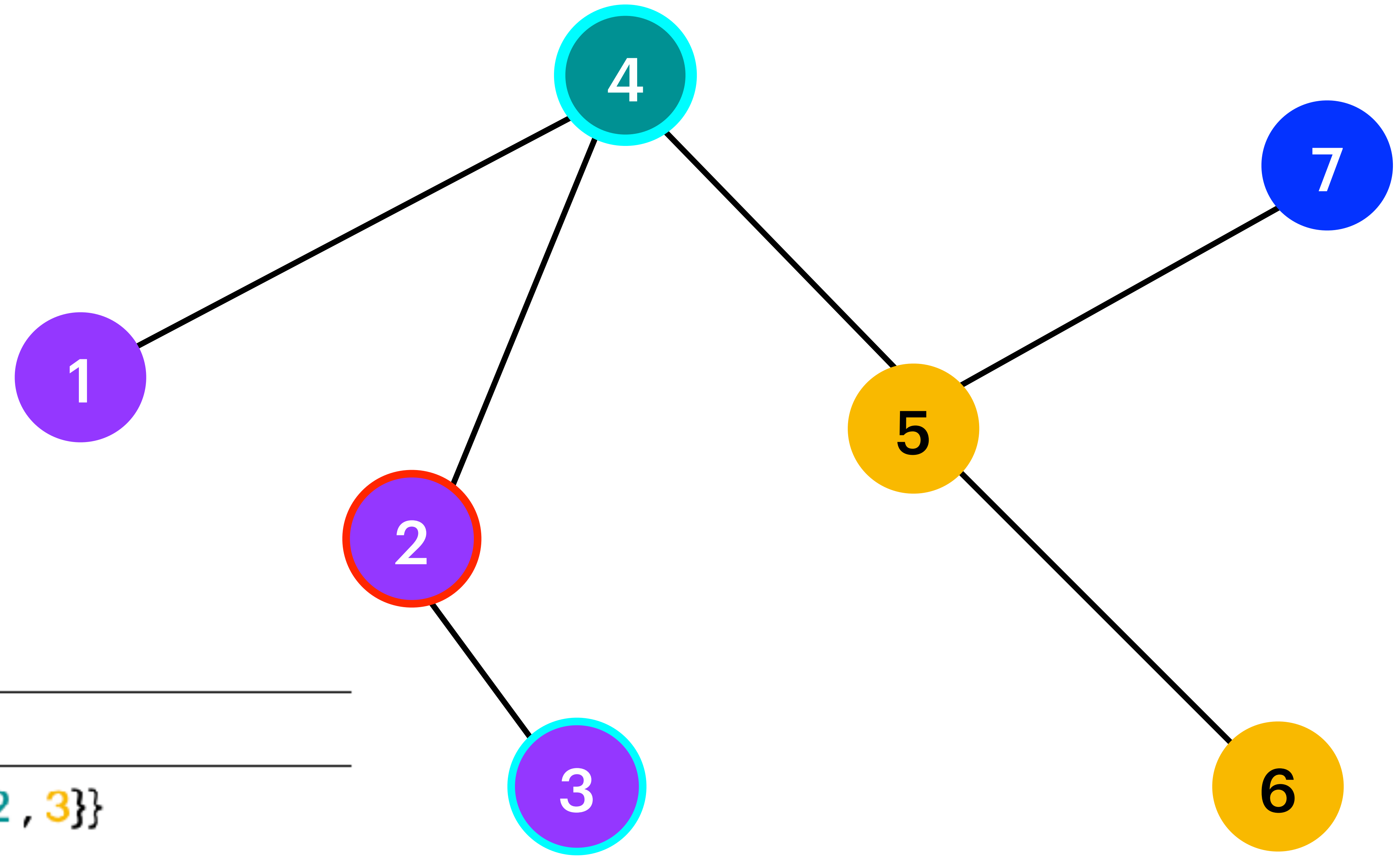
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_3	
$P_3(1)$	{{1 , 2 , 3 , 4}}
$P_3(2)$	{{1 , 2 , 3 , 4}}
$P_3(3)$	
$P_3(4)$	
$P_3(5)$	
$P_3(6)$	
$P_3(7)$	



P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	{{1 , 2 , 3}}
$P_2(6)$	∅
$P_2(7)$	{{2 , 3 , 4}}

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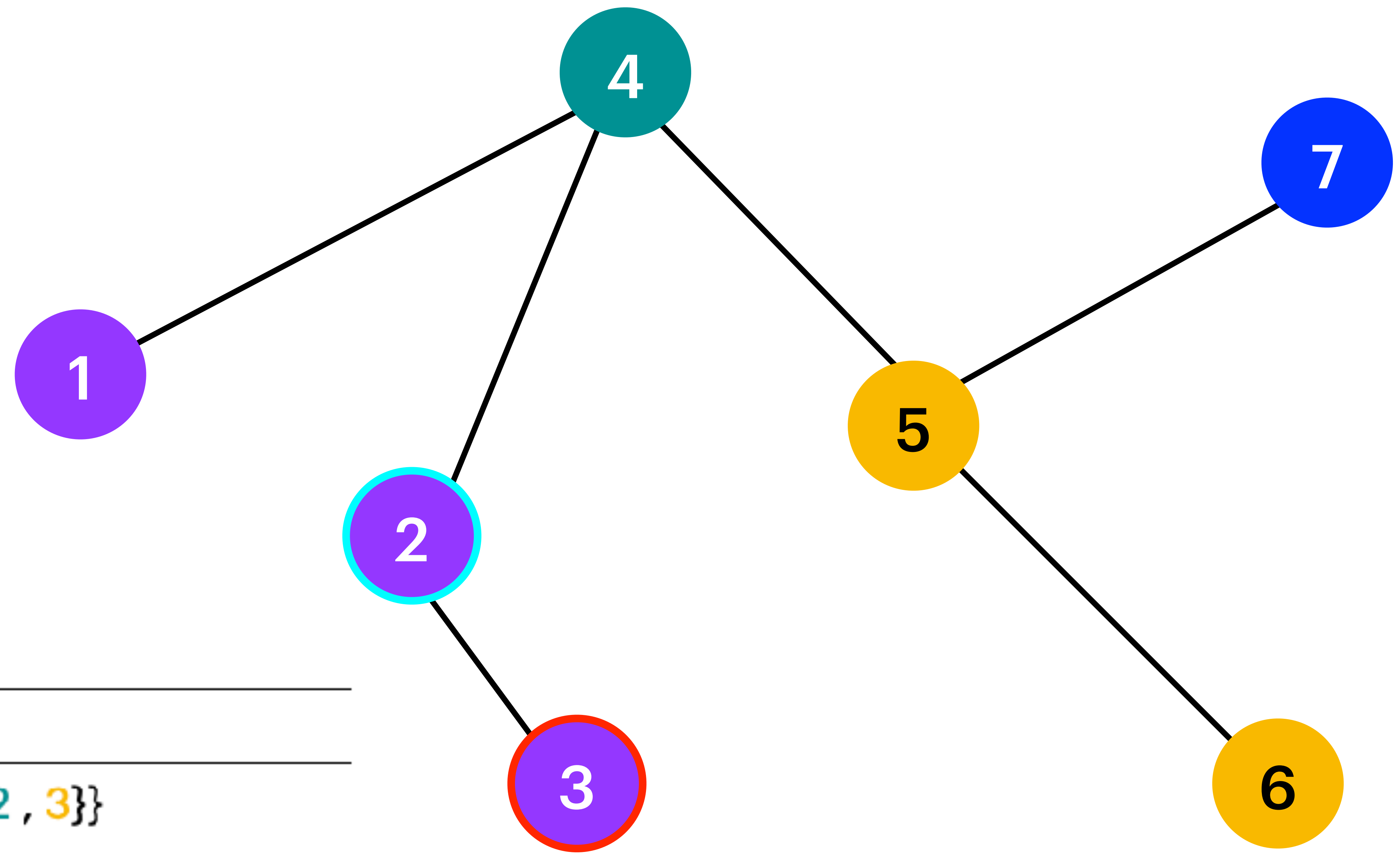
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_3	
$P_3(1)$	{{1 , 2 , 3 , 4}}
$P_3(2)$	{{1 , 2 , 3 , 4}}
$P_3(3)$	
$P_3(4)$	
$P_3(5)$	
$P_3(6)$	
$P_3(7)$	



P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	Ø
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	{{1 , 2 , 3}}
$P_2(6)$	Ø
$P_2(7)$	{{2 , 3 , 4}}

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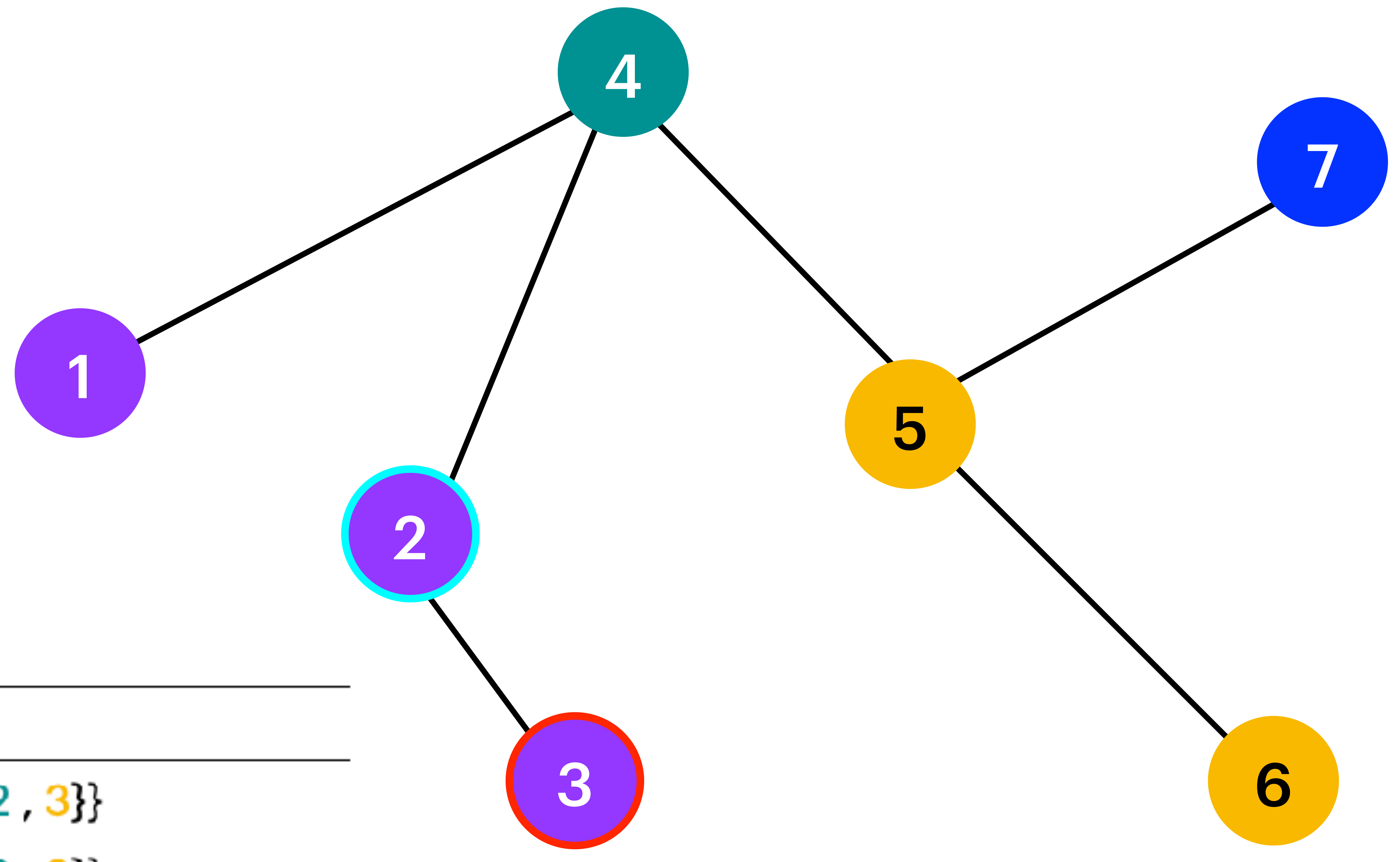
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_3	
$P_3(1)$	{{1 , 2 , 3 , 4}}
$P_3(2)$	{{1 , 2 , 3 , 4}}
$P_3(3)$	∅
$P_3(4)$	
$P_3(5)$	
$P_3(6)$	
$P_3(7)$	



P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	{{1 , 2 , 3}}
$P_2(6)$	∅
$P_2(7)$	{{2 , 3 , 4}}

γ 1 , 2 , 3 , 4

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

Colorful Paths

Algorithm

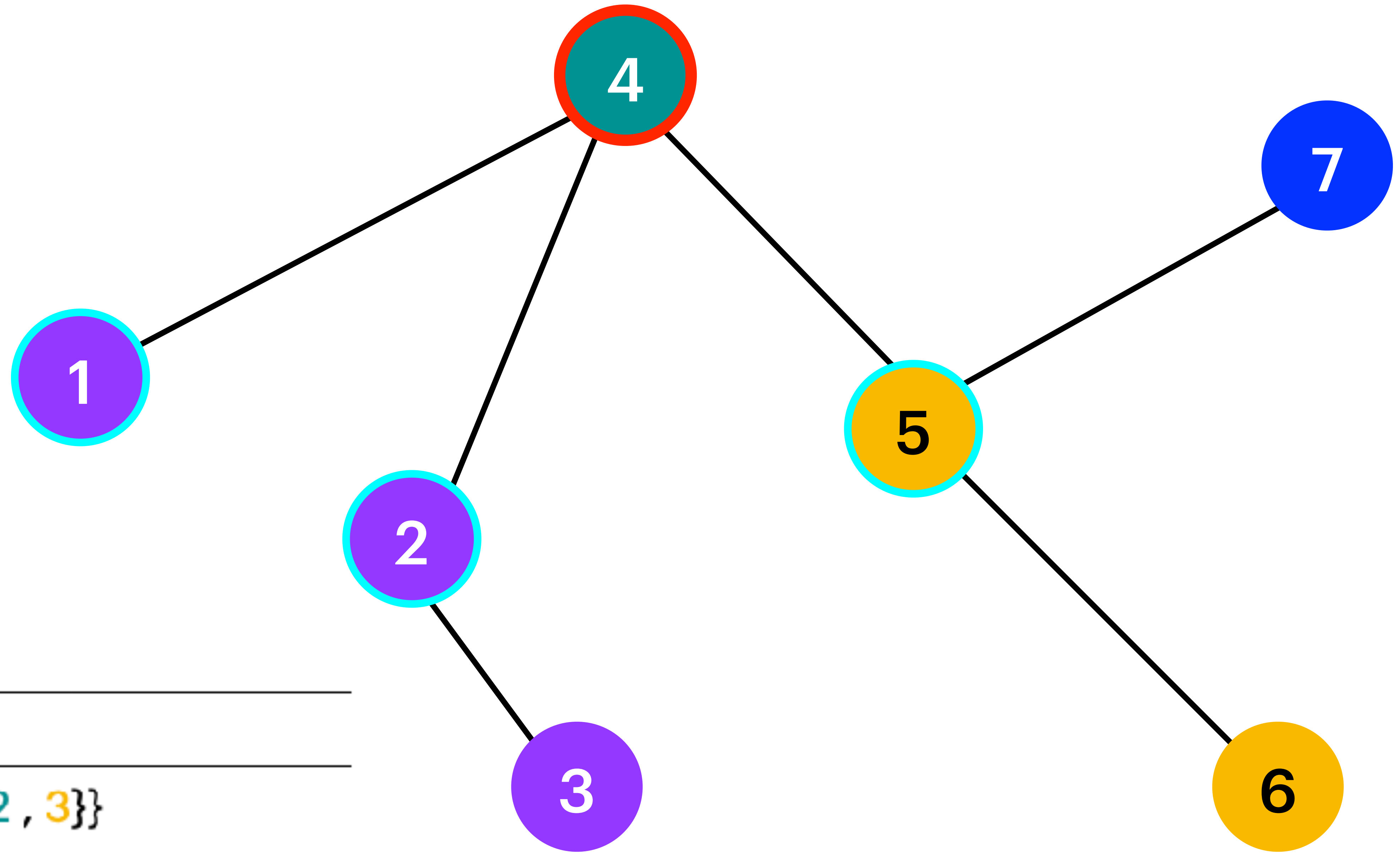
Algorithm 2: RAINBOW(G, γ)

- **forall** $v \in V$ **do**
  - $P_0(v) \leftarrow \{\{\gamma(v)\}\}$;
- **for** $i = 1$ **to** $k-1$ **do**
  - COLORFUL(G, i);
- **return** $\bigcup_{v \in V} P_{k-1}(v) \neq \emptyset$;

Algorithm 1: COLORFUL(G, i)

- **forall** $v \in V$ **do**
  - $P_i(v) \leftarrow \emptyset$;
  - **forall** $x \in N(v)$ **do**
    - **forall** $R \in P_{i-1}(x)$ *such that* $\gamma(v) \notin R$ **do**
      - $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

P_3	
$P_3(1)$	$\{\{1, 2, 3, 4\}\}$
$P_3(2)$	$\{\{1, 2, 3, 4\}\}$
$P_3(3)$	$\emptyset$
$P_3(4)$	
$P_3(5)$	
$P_3(6)$	
$P_3(7)$	



P_2	
$P_2(1)$	$\{\{1, 2, 3\}\}$
$P_2(2)$	$\{\{1, 2, 3\}\}$
$P_2(3)$	$\emptyset$
$P_2(4)$	$\{\{2, 3, 4\}\}$
$P_2(5)$	$\{\{1, 2, 3\}\}$
$P_2(6)$	$\emptyset$
$P_2(7)$	$\{\{2, 3, 4\}\}$

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

γ 1 , 2 , 3 , 4

Colorful Paths
Algorithm

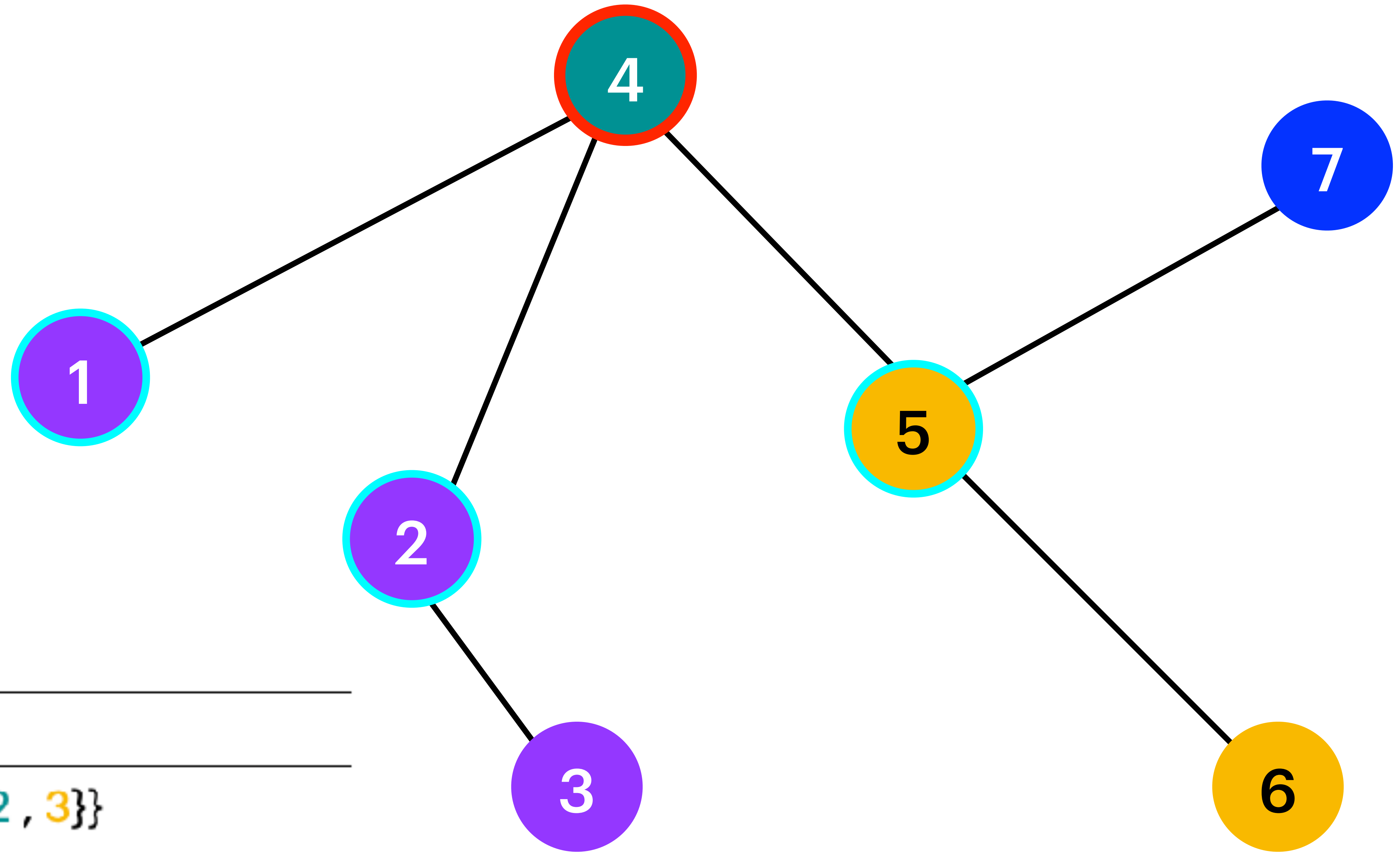
Algorithm 2: $\textsc{Rainbow}(G, \gamma)$

$$
\begin{aligned}
&\textbf{forall } v \in V \textbf{ do}\\
&\quad P_0(v) \leftarrow \{\{\gamma(v)\}\};\\
&\textbf{for } i = 1 \textbf{ to } k-1 \textbf{ do}\\
&\quad \textsc{Colorful}(G, i);\\
&\textbf{return } \bigcup_{v \in V} P_{k-1}(v) \neq \emptyset;
\end{aligned}
$$

Algorithm 1: $\textsc{Colorful}(G, i)$

$$
\begin{aligned}
&\textbf{forall } v \in V \textbf{ do}\\
&\quad P_i(v) \leftarrow \emptyset;\\
&\quad \textbf{forall } x \in N(v) \textbf{ do}\\
&\quad\quad \textbf{forall } R \in P_{i-1}(x) \textit{ such that } \gamma(v) \notin R \textbf{ do}\\
&\quad\quad\quad P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\};
\end{aligned}
$$

P_3	
$P_3(1)$	{{1, 2, 3, 4}}
$P_3(2)$	{{1, 2, 3, 4}}
$P_3(3)$	∅
$P_3(4)$	∅
$P_3(5)$	
$P_3(6)$	
$P_3(7)$	



P_2	
$P_2(1)$	{{1, 2, 3}}
$P_2(2)$	{{1, 2, 3}}
$P_2(3)$	∅
$P_2(4)$	{{2, 3, 4}}
$P_2(5)$	{{1, 2, 3}}
$P_2(6)$	∅
$P_2(7)$	{{2, 3, 4}}

γ 1, 2, 3, 4

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

Colorful Paths

Algorithm

Algorithm 2: RAINBOW(G, γ)

forall $v \in V$ **do**
  $P_0(v) \leftarrow \{\{\gamma(v)\}\}$;
for $i = 1$ **to** $k-1$ **do**
  COLORFUL(G, i);
return $\bigcup_{v \in V} P_{k-1}(v) \neq \emptyset$;

Algorithm 1: COLORFUL(G, i)

forall $v \in V$ **do**
  $P_i(v) \leftarrow \emptyset$;
  forall $x \in N(v)$ **do**
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P_3	
$P_3(1)$	{{1 , 2 , 3 , 4}}
$P_3(2)$	{{1 , 2 , 3 , 4}}
$P_3(3)$	∅
$P_3(4)$	∅
$P_3(5)$	
$P_3(6)$	
$P_3(7)$	

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

P_2	
$P_2(1)$	{{1 , 2 , 3}}
$P_2(2)$	{{1 , 2 , 3}}
$P_2(3)$	∅
$P_2(4)$	{{2 , 3 , 4}}
$P_2(5)$	{{1 , 2 , 3}}
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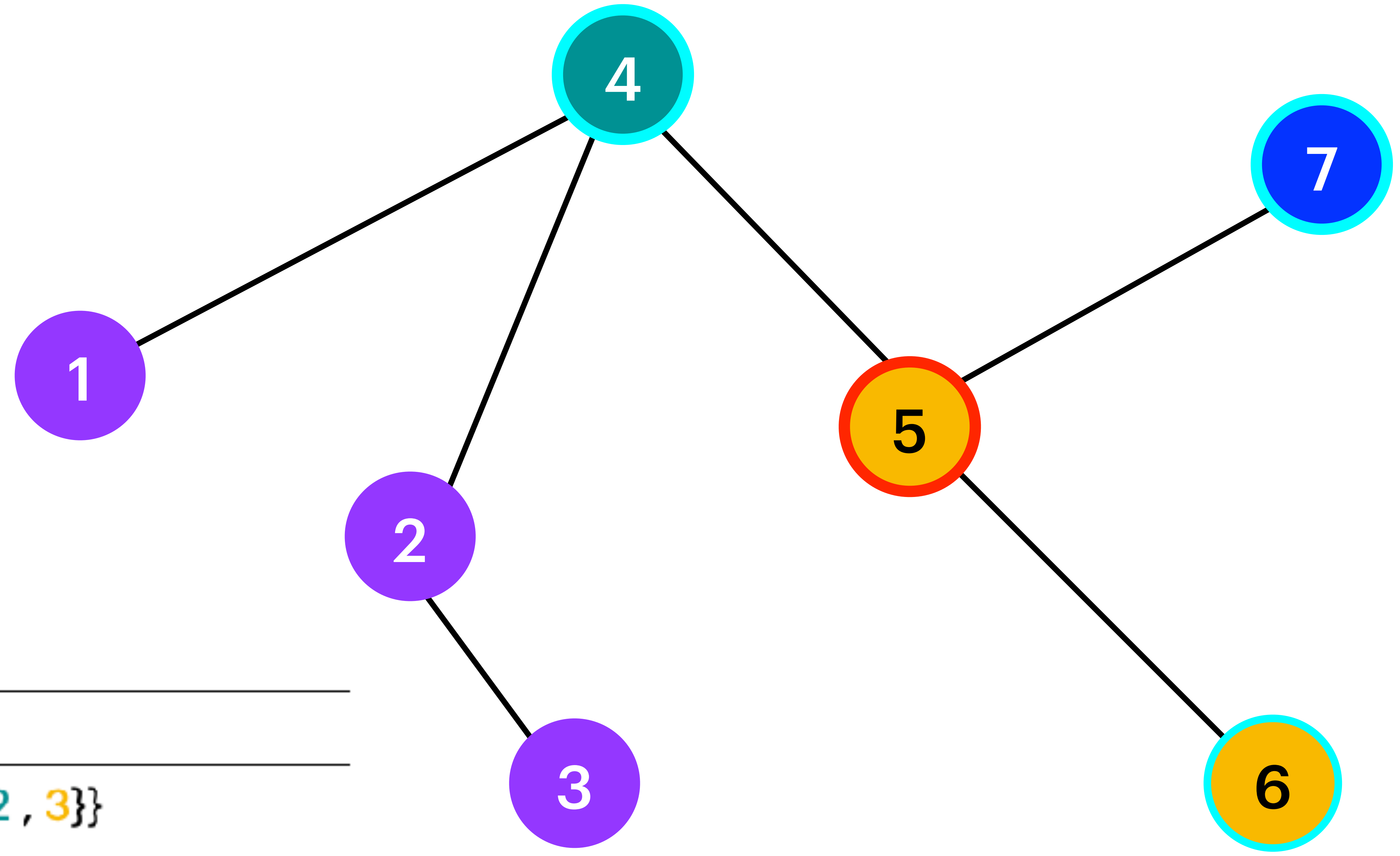

γ 1 , 2 , 3 , 4

Colorful Paths

Algorithm

Algorithm 2: RAINBOW(G, γ)

```
forall v ∈ V do
    P_0(v) ← {{γ(v)}};
for i = 1 to k − 1 do
    COLORFUL(G, i);
return ⋃_{v∈V} P_{k−1}(v) ≠ ∅;
```

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forall v ∈ V do
    P_i(v) ← ∅;
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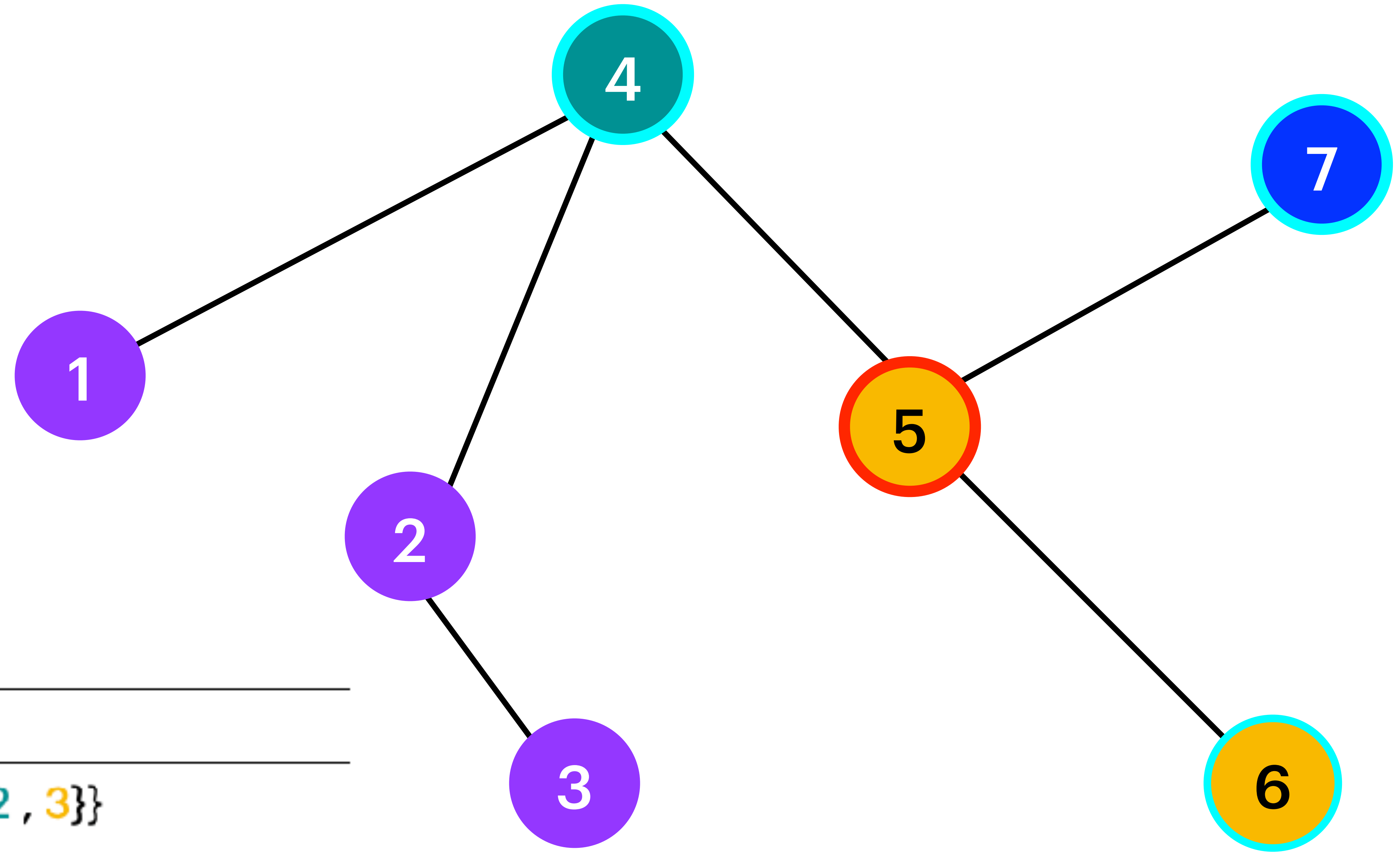

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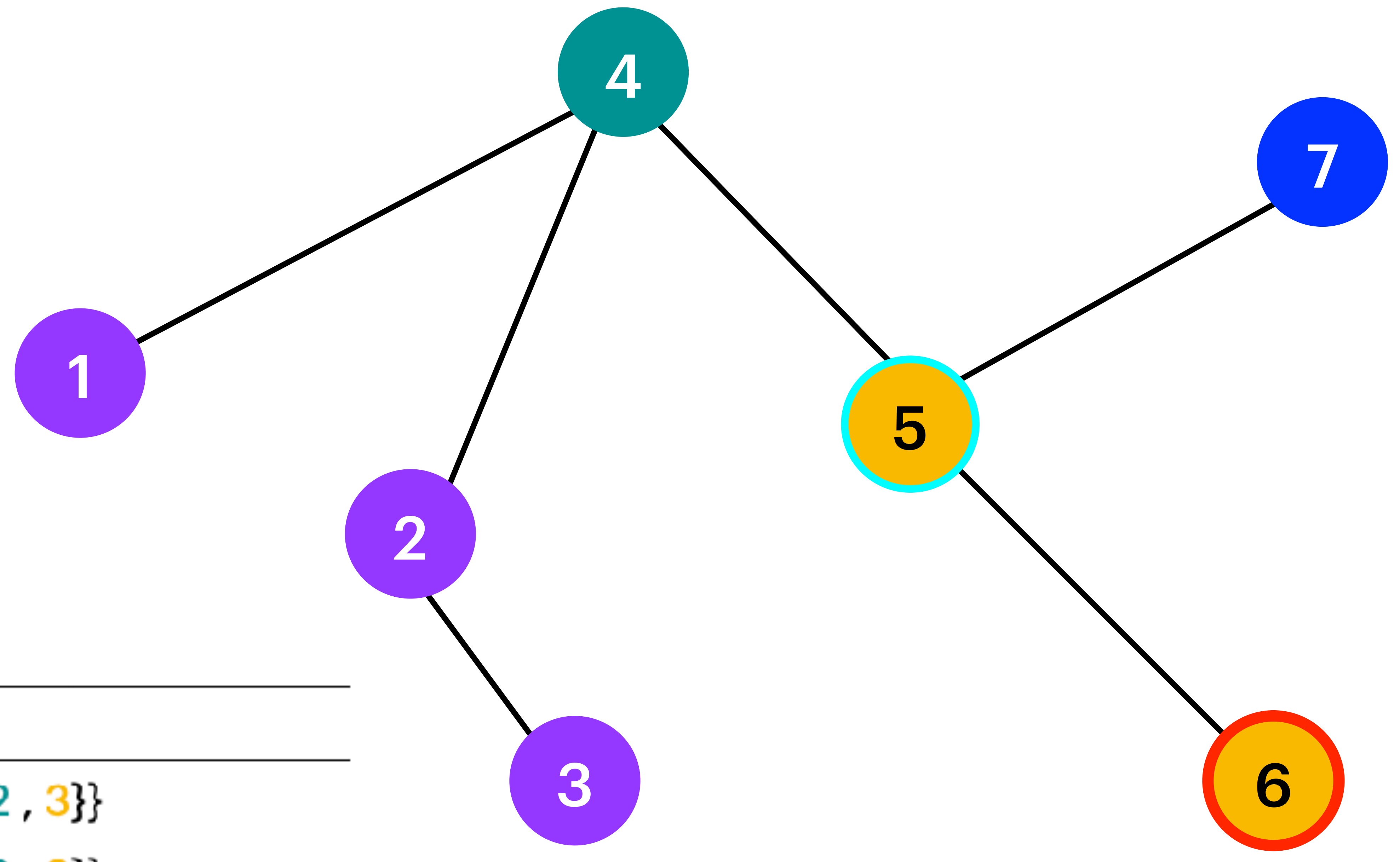

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Colorful Paths

Algorithm

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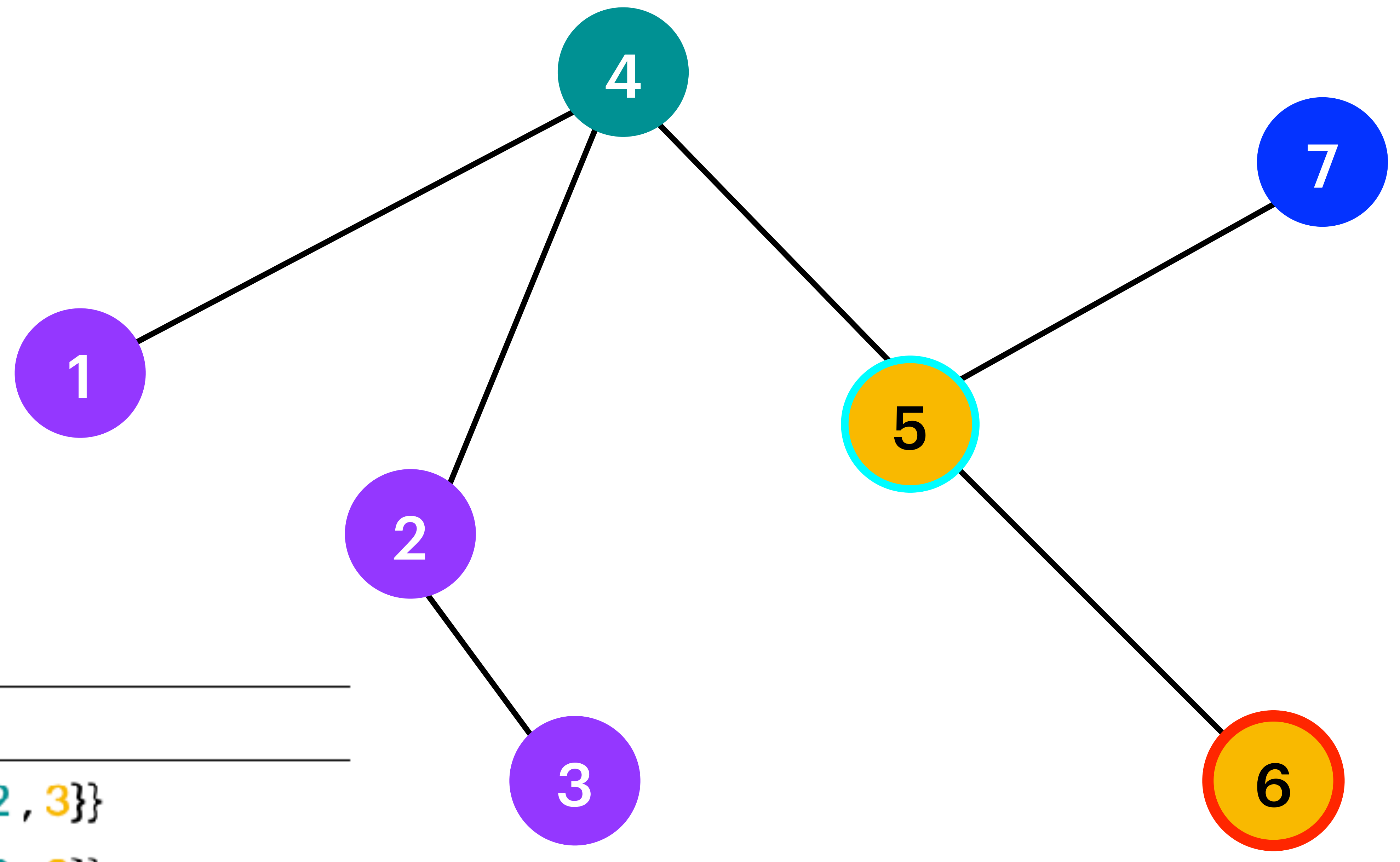

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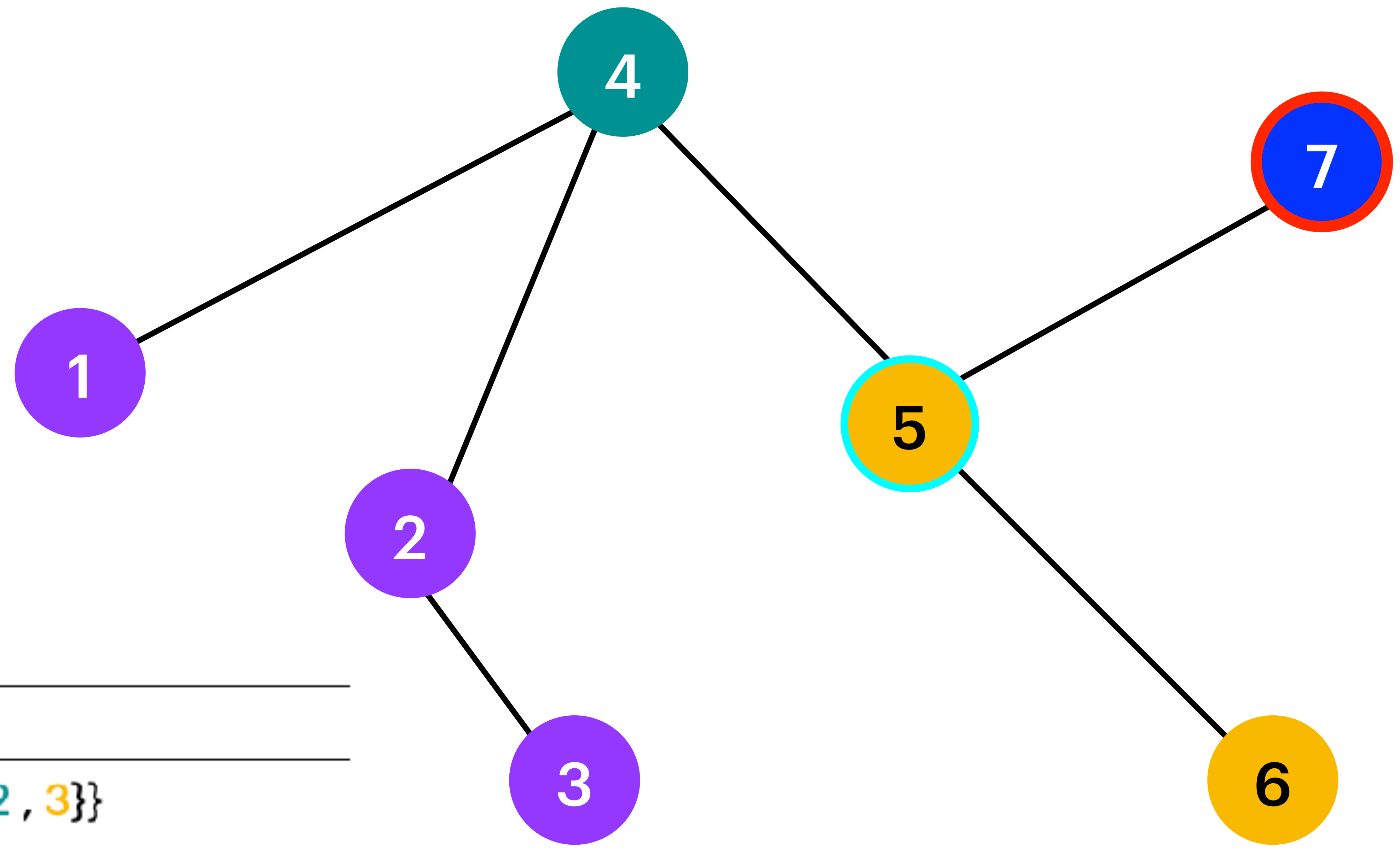

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Colorful Paths

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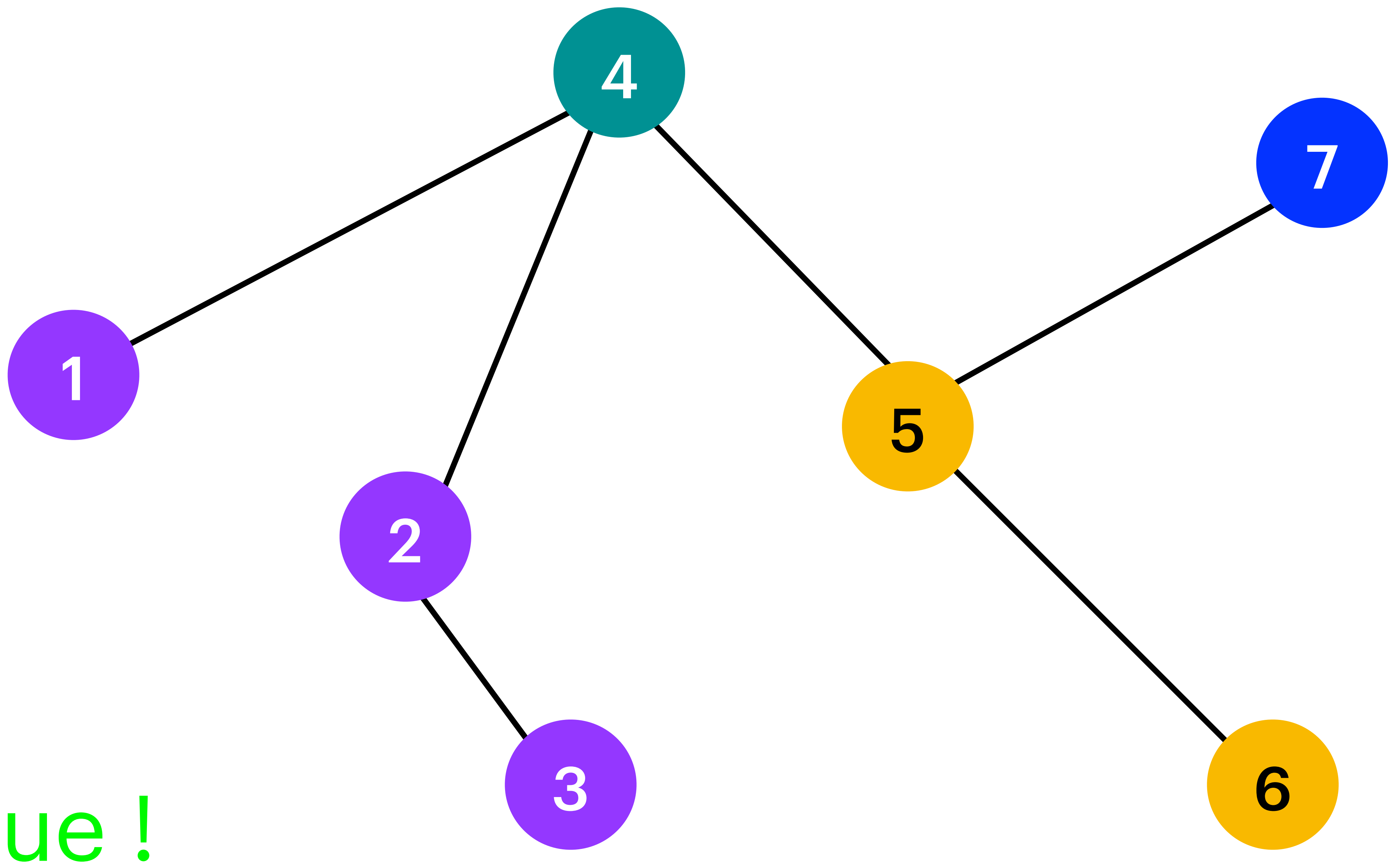

returns true !

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

γ 1, 2, 3, 4

Colorful Paths

Algorithm

Algorithm 2: RAINBOW(G, γ)

1 **forall** $v \in V$ **do**
2 $\quad P_0(v) \leftarrow \{\{\gamma(v)\}\}$;
3 **for** $i = 1$ **to** $k - 1$ **do**
4 $\quad$ COLORFUL(G, i);
5 **return** $\bigcup_{v \in V} P_{k-1}(v) \neq \emptyset$;

Algorithm 1: COLORFUL(G, i)

1 **forall** $v \in V$ **do**
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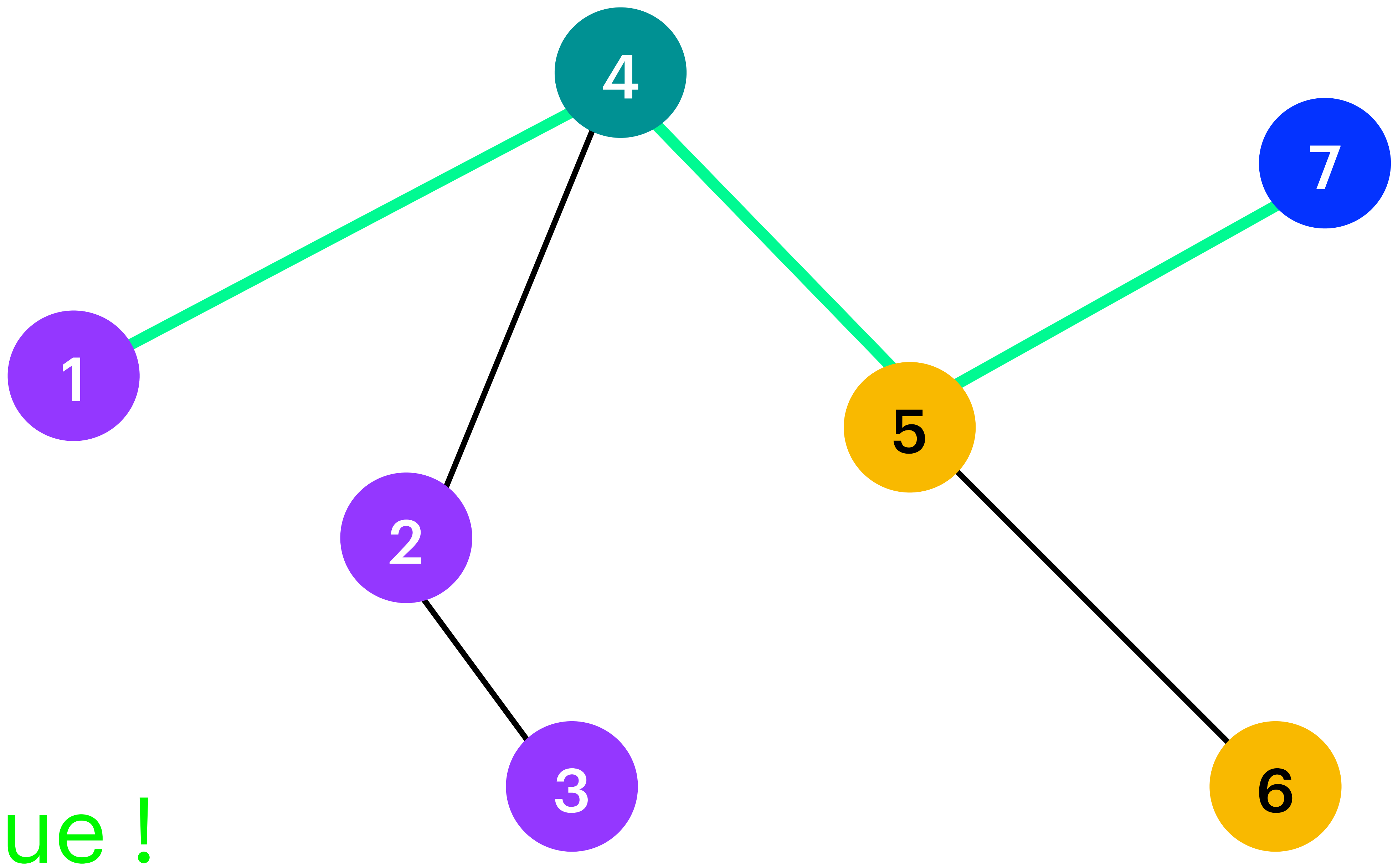

returns true !

color sets S s.t |S| = i+1 and there is a colorful path of length i ending at v only using the colors in S

given : A graph $G = (V, E)$

A coloring of its vertices with k colors $\gamma : V \rightarrow [k]$

to find : Does there exist a colorful path of length $k - 1$ in a randomly colored graph ? A path is colorful if all of the vertices in the path have a different color

$\exists$ colorful path of length $k-1 \iff \bigcup_{v \in V} P_{k-1}(v) \neq \varnothing$

1 , 2 , 3 , 4

Colorful Paths

Algorithm + Probability

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$\exists$ colorful path of length $k-1 \iff \bigcup_{v \in V} P_{k-1}(v) \neq \varnothing$

$$\mathcal{O}(2^k \cdot k \cdot m)$$

Algorithm 1: COLORFUL(G, i) — G a γ-colored graph

forall $v \in V$ **do**
  $P_i(v) \leftarrow \emptyset$;
  forall $x \in N(v)$ **do**
    forall $R \in P_{i-1}(x)$ *such that* $\gamma(v) \notin R$ **do**
      $P_i(v) \leftarrow P_i(v) \cup \{R \cup \{\gamma(v)\}\}$;

Algorithm 2: RAINBOW(G, γ) — G a graph, γ a k-coloring

forall $v \in V$ **do**
  $P_0(v) \leftarrow \{\{\gamma(v)\}\}$;
for $i = 1$ **to** $k-1$ **do**
  COLORFUL(G, i);
return $\bigcup_{v \in V} P_{k-1}(v) \neq \emptyset$;

Satz 3.2

Let G be a graph that contains a path of length $k-1$.

1. A random coloring of the graph using k colors produces a colorful path of length $k-1$ with probability at least

$$p_{\text{success}} \geq \frac{k!}{k^k} \geq e^{-k}$$

2. If we repeat the coloring process multiple times, then the **expected number of repetitions** needed until success is at most

$$\frac{1}{p_{\text{success}}} \leq e^k$$

Satz 3.3

1. The full randomized algorithm (including repetitions) has a runtime of

$$\mathcal{O}(\lambda \cdot (2e)^k \cdot km)$$

where $\lambda > 1$ is a tunable parameter (confidence level).

2. If the algorithm answers **YES**, then the graph **does contain** a path of length $k-1$.
3. If the graph does contain a path of length $k-1$, then the probability that the algorithm answers **NO** is at most

$$e^{-\lambda}$$

Last Weeks ...

- 21.05 session : Minimum Cut, Smallest Enclosing Cycle

- **26.05 END OF SEMESTER, WHAT NOW? (18:00 - 21:00)**
- 28.05 session : Convex Hull

- Last extra session : Coding
    - worst case zoom

Questions

Feedbacks , Recommendations

Nil Ozer

Helper

Mathematical Tools and Notations

$[n] := \{1,2,\ldots,n\}$

$[n]^k :=$ the set of sequences over $[n]$ of length k $\quad |[n]^k| = n^k$

$\binom{[n]}{k} :=$ the set of k-element subsets of $[n]$ $\quad \left|\binom{[n]}{k}\right| = \binom{n}{k}$.

The k nodes on a path of length $k-1$ can be colored using $[k]$ in exactly k^k ways

$k!$ of these colorings use each color exactly once

Helper

Mathematical Tools and Notations

Handshaking lemma : For all graphs , it holds that $\sum_{v \in V} \deg(v) = 2|E|$.

If you repeat an experiment with success probability p until success, then the expected number of trials is $\frac{1}{p}$ ($Geo(p)$)

Helper

Mathematical Tools and Notations

For $c, n \in \mathbb{R}^+$, it holds that $c^{\log n} = n^{\log c}$

$$2^{\log n} = n^{\log 2} = n \text{ and } 2^{\mathcal{O}(\log n)} = n^{\mathcal{O}(1)} \text{ is always polynomial in } n$$

For $n \in \mathbb{N}_0$, it holds that $\sum_{i=0}^{n} \binom{n}{i} = 2^n$ (binomial theorem)

For $n \in \mathbb{N}_0$, it holds that $\frac{n!}{n^n} \geq e^{-n}$ (power series expansion of the exponential function)